Lecture 15

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1. (a) $(a,b) \in R \iff a < 3b$

   Reflexive. (not antisymmetric because $(2,1),(1,2) \in R$, not transitive because $(9,5),(5,3) \in R$ but $(9,3) \notin R$)

   (b) $(a,b) \in R \iff a - b = 0$

   Reflexive, symmetric, transitive, antisymmetric.

   (c) $(a,b) \in R \iff 3a \leq 2b$

   Irreflexive, transitive ($3a \leq 2b \wedge 3b \leq 2c \implies 3a \leq 2b \leq 3b \leq 2c \implies 3a \leq 2c$), antisymmetric (if $(a,b),(b,a) \in R$ then by transitivity $(a,a) \in R$ which contradicts irreflexivity, therefore $(a,b),(b,a) \in R$ never holds).

   (d) $(a,b) \in R \iff 7|(3a+4b)$

   Note that $3a + 4b \equiv 0 \pmod 7 \iff 3a \equiv -4b \pmod 7 \iff 3a \equiv 3b \pmod 7 \iff a \equiv b \pmod 7$. Therefore, it is reflexive, symmetric, transitive.

2. Consider the set $S = [4]$ with the following relations

   (a) $R_1 = \{(1,3),(2,4),(3,1),(4,2)\}$: Irreflexive, symmetric.

   (b) $R_2 = \{(1,3),(2,4)\}$: Irreflexive, antisymmetric, transitive.

   (c) $R_3 = \{(1,1),(2,2),(3,3),(4,4),(1,3),(2,4),(3,1),(4,2)\}$: Reflexive, symmetric, transitive.

   (d) $R_4 = \{(1,1),(2,2),(3,3),(4,4)\}$: Reflexive, symmetric, transitive, antisymmetric.

   (e) $R_5 = \{(1,1),(2,2),(3,3),(4,4),(1,2),(2,3),(3,4),(4,3),(3,2),(2,1)\}$: Reflexive, symmetric.

3. $A = [5]$

   (a) Example of relation that is reflexive but not transitive:

   $$R = \{(1,1),(2,2),(3,3),(4,4),(5,5),(1,2),(2,3)\}$$

   (b) Example of a relation that is reflexive and symmetric but not transitive:

   $$R = \{(1,1),(2,2),(3,3),(4,4),(5,5),(1,2),(2,1),(2,3),(3,2)\}$$

   (c) Example of a relation that is irreflexive and transitive.

   $$R = \varnothing$$

4. Consider the relation R defined on $\mathbb{R} \times \mathbb{R}$ by $(x,y)R(x',y')$ iff $x - x' \in \mathbb{Z} \wedge y - y' \in \mathbb{Z}$. Prove that R is reflexive, symmetric and transitive.

   Reflexive: $x - x = 0 \in \mathbb{Z} \wedge y - y = 0 \in \mathbb{Z}$ so $(x,y)R(x,y)$.

   Symmetric: Suppose $(x,y)R(x',y')$, then $x' - x = -(x - x') \in \mathbb{Z} \wedge y' - y = -(y - y') \in \mathbb{Z}$ so $(x',y')R(x,y)$.

   Transitive: Suppose $(x,y)R(x',y')$ and $(x',y')R(x'',y'')$, then $x - x'' = (x - x') + (x' - x'') \in \mathbb{Z}$ and $y - y'' = (y - y') + (y' - y'') \in \mathbb{Z}$ so $(x,y)R(x'',y'')$.