Lecture 10

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October 1, 2013

1. Write each of the following mathematical satements in symbolic form and then determine wether it is true of false.
   (a) There is a real number that is strictly bigger than every integer.
   $\exists x \in \mathbb{R}.\ \forall n \in \mathbb{Z}.\ x > n$ False
   (b) Every subset of $\mathbb{N}$ has the number 3 as an element.
   $\forall S \subseteq \mathbb{N}.\ 3 \in S$ False
   (c) Every natural number's square root is a real number.
   $\forall n \in \mathbb{N}.\ \sqrt{n} \in \mathbb{R}$ Or alternatively
   $\forall n \in \mathbb{N}.\ \exists x \in \mathbb{R}.\ x^2 = n$ True
2. For each of the following quantified statements, say it out load by reading the symbolic notation. Then determine wether the statement is true or false.
   (a) $\forall x \in \mathbb{N}.\ \exists y \in \mathbb{Z}.\ x + y < 0$ True
   (b) $\exists y \in \mathbb{Z}.\ \forall x \in \mathbb{N}.\ x + y < 0$ False
   (c) $\forall a \in \mathbb{N}.\ \exists b \in \mathbb{Z}.\ \forall c \in \mathbb{N}.\ a + b < c$ True
3. Write the logical negation of the statements in the previous problem.
   (a) $\exists x \in \mathbb{N}.\ \forall y \in \mathbb{Z}.\ x + y \geq 0$
   (b) $\forall y \in \mathbb{Z}.\ \exists x \in \mathbb{N}.\ x + y \geq 0$
   (c) $\exists a \in \mathbb{N}.\ \forall b \in \mathbb{Z}.\ \exists c \in \mathbb{N}.\ a + b \geq c$
4. Define the set V below using set builder notation and quantifiers.
$$V = \bigcup_{x \in I} \{y \in \mathbb{R} \mid -3x < y < 4x\}$$
$$V = \{y \in \mathbb{R} \mid \exists x \in I. -3x < y < 4x\}$$
5. Let's define the following variable propositions:
$$P(x) \text{ is } \text{“}\frac{1}{2} < x\text{”} \qquad Q(x) \text{ is } \text{“}x < \frac{3}{2}\text{”}$$
$$R(x) \text{ is } \text{“}x^2 = 4\text{”} \qquad S(x) \text{ is } \text{“}x + 1 \in \mathbb{N}\text{”}$$
   (a) $\forall x \in \mathbb{N}.\ P(x)$ True
   (b) $\forall x \in \mathbb{N}.\ Q(x) \implies P(x)$ True
   (c) $\forall x \in \mathbb{Z}.\ Q(x) \implies P(x)$ False
   (d) $\forall x \in \mathbb{R}.\ R(x) \implies S(x)$ False
6. (See Example 4.6.5 (p.264) for more details) Let $P(x)$ be the proposition "x is an integer that is divisible by 6". For each of the following conditions, identify whether it is **necessary** or **sufficient** condition for $P(x)$ to hold.
   (a) Let $Q(x)$ be "x is an integer that is divisible by 3."
   $Q(x)$ is necessary but not sufficient, so $\forall x \in \mathbb{Z}. P(x) \implies Q(x)$
   (b) Let $R(x)$ be "x is an integer that is divisible by 12."
   $R(x)$ is sufficient but not necessary, so $\forall x \in \mathbb{Z}. R(x) \implies P(x)$
   (c) Let $S(x)$ be "x is an integer such that x^2 is divisble by 6"
   $S(x)$ is necessary and sufficient, so $\forall x \in \mathbb{Z} P(x) \iff S(x)$