Lecture 19

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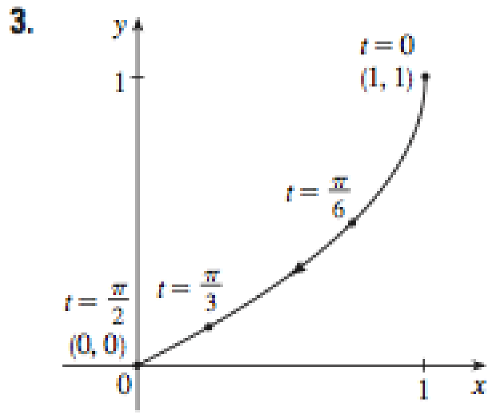
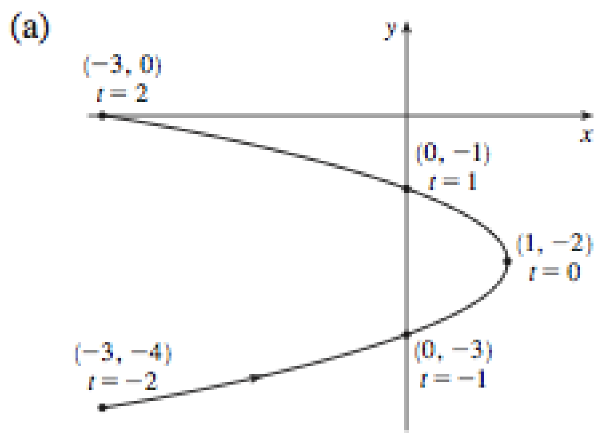
1. Sketch the curve $x = \cos^2 t,\ y = 1 - \sin t,\ 0 \le t \le \pi/2$.



2. Sketch the curve $x = 1 - t^2,\ y = t - 2,\ -2 \le t \le 2$.



   Eliminate the parameter to find a Cartesian equation for the curve. Use $t = y + 2$ and substitute $x = 1 - (y + 2)^2$, and $-4 \le y \le 0$.

3. Describe the motion of a particle with position (x, y) as t varies in the given interval. $x = 3 + 2\cos t,\ y = 1 + 2\sin t,\ \pi/2 \le t \le 3\pi/2$.

   From the parametric equation, we can see that it is a circle centered around $(3, 1)$ with radius 2, which has equation $(x - 3)^2 + (y - 1)^2 = 4$. Also, notice that we are moving counterclockwise. At $t = \pi/2$, we are at $(3, 3)$, and at $t = 3\pi/2$, we are at $(3, -1)$. Also, note that since the interval of t has width π, so we make less than one revolution.

   Therefore, the particle moves counterclockwise along the circle $(x-3)^2 + (y-1)^2 = 4$, moving from $(3, 3)$ to $(3, -1)$.

4. Find an equation of the tangent to the curve $x = 1 + 4t - t^2,\ y = 2 - t^3$ at $t = 1$.

   At $t = 1$, $x = 1 + 4 - 1 = 4$ and $y = 2 - 1 = 1$. Also, $\frac{dx}{dt} = 4 - 2t$, and $\frac{dy}{dt} = -3t^2$.

   Thus, at $t = 1$,

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-3(1)^2}{4 - 2(1)} = -\frac{3}{2}$$

thus the tangent line is $y = -\frac{3}{2}(x-4)+1$.

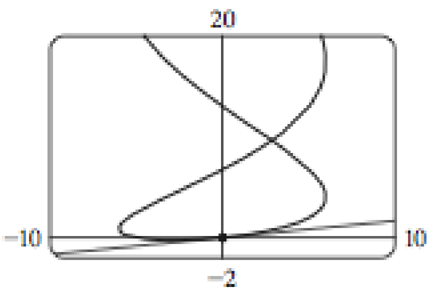
5. Find the equation of the tangent(s) to the curve $x = 6\sin t,\ y = t^2 + t$ at the point $(0, 0)$.

   Notice $0 = x = 6\sin t \implies t = k\pi$ for some $k \in \mathbb{Z}$. Also $0 = y = t^2 + t = (t+1)t$ so $t = -1, 0$. Therefore, the only possible t is $t = 0$.

   Then since $\frac{dx}{dt} = 6\cos t$ and $\frac{dy}{dt} = 2t + 1$, so

$$\frac{dy}{dx} = \frac{2(0)+1}{6\cos 0} = \frac{1}{6}$$

   so the tangent line is $y = \frac{1}{6}(x-0)+0$ so $y = \frac{1}{6}x$.



6. At what points on the curve $x = 2t^3$ and $y = 1 + 4t - t^2$ does the tangent line have slope 1?

   First $\frac{dx}{dt} = 6t^2$ and $\frac{dy}{dt} = 4 - 2t$ so when $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 1$, we know that

$$\begin{aligned} 6t^2 &= 4 - 2t \\ 6t^2 + 2t - 4 &= 0 \\ (2t+2)(3t-2) &= 0 \end{aligned}$$

   so $t = -1, \frac{2}{3}$.

   At $t = -1$, $x = 2(-1)^3 = -2$ and $y = 1 + 4(-1) - (-1)^2 = -4$.

   At $t = \frac{2}{3}$, $x = 2(\frac{2}{3})^3 = \frac{16}{27}$ and $y = 1 + 4(\frac{2}{3}) - (\frac{2}{3})^2 = 1 + \frac{8}{3} - \frac{4}{9} = \frac{9}{9} + \frac{24}{9} - \frac{4}{9} = \frac{29}{9}$.

   Thus the points are $(-2, -4)$ and $(\frac{16}{27}, \frac{29}{9})$.