Lecture 14

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1.
$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n}}$$

Use Integral Test

$$\int_2^{\infty} \frac{1}{x\sqrt{x}}dx = \lim_{t\to\infty} \int_{x=2}^{x=t} u^{-\frac{1}{2}}dx = \lim_{t\to\infty} 2\sqrt{u}\Big]_{x=2}^{x=t} = \lim_{t\to\infty} 2\sqrt{\ln x}\Big]_2^t = \infty$$

using $u = \ln x,\ du = \frac{1}{x}dx$. So by Integral Test it series diverges.

2.
$$\sum_{k=1}^{\infty} k^2 e^{-k}$$

Use Ratio Test

$$\lim_{k\to\infty} |\frac{a_{k+1}}{a_k}| = \lim_{k\to\infty} \frac{(k+1)^2 e^{-k-1}}{k^2 e^{-k}} = \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^2 \frac{1}{e} = \frac{1}{e} < 1$$

absolutely converges.

3.
$$\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdots(2n-1)}{2\cdot 5\cdot 8\cdots(3n-1)}$$

Use Ratio Test

$$\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = \lim_{n\to\infty} \frac{1\cdot 3\cdots(2n+1)}{2\cdot 5\cdots(3n+2)} \frac{2\cdot 5\cdots(3n-1)}{1\cdot 3\cdots(2n-1)} = \lim_{n\to\infty} \frac{2n+1}{3n+2} = \frac{2}{3} < 1$$

absolutely converges.

4.
$$\sum_{n=1}^{\infty} (-1)^n \cos(\frac{1}{n^2})$$

Alternating Series. Need to check $b_{n+1} \le b_n$ and $\lim_{n\to\infty} \cos(\frac{1}{n^2}) = 0$

Let $f(x) = \cos(\frac{1}{x^2})$, then $f'(x) = \frac{2}{x^3}\sin(\frac{1}{x^2})$, so for $x \ge 1$, $f'(x) > 0$ so in fact the b_n's are increasing so the series diverges.

Alternatively, we first check $\lim_{n\to\infty} \cos(\frac{1}{n^2}) = \cos(0) = 1$ so this also fails so the series diverges.

5.
$$\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$$

Use Ratio Test

$$\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = \lim_{n\to\infty} \frac{(n+1)!}{e^{(n+1)^2}} \frac{e^{n^2}}{n!} = \lim_{n\to\infty} \frac{n+1}{e^{2n+1}} = \lim_{n\to\infty} \frac{1}{2e^{2n+1}} = 0$$

by L'Hoptial's Rule.