Lecture 11

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1.
$$\frac{1}{3}+\frac{1}{6}+\frac{1}{9}+\frac{1}{12}+\frac{1}{15}+\cdots=\sum_{n=1}^{\infty}\frac{1}{3n}=\frac{1}{3}\sum_{n=1}^{\infty}\frac{1}{n}$$
so it diverges (by p-test).

2.
$$\sum_{n=1}^{\infty}\frac{1+2^n}{3^n}=\sum_{n=1}^{\infty}(\frac{1}{3})^n+\sum_{n=1}^{\infty}(\frac{2}{3})^n=\left(\sum_{n=0}^{\infty}(\frac{1}{3})^n\right)-1+\left(\sum_{n=0}^{\infty}(\frac{2}{3})^n\right)-1=\frac{1}{1-\frac{1}{3}}+\frac{1}{1-\frac{2}{3}}-2=\frac{3}{2}+\frac{3}{1}-2=\frac{5}{2}$$
using $\sum_{n=1}^{\infty}ar^{n-1}=\sum_{n=0}^{\infty}ar^n=\frac{a}{1-r}$ for $|r|<1$.

3.
$$\sum_{k=0}^{\infty}(\frac{\pi}{3})^k$$
diverges since $|\frac{\pi}{3}|>1$ since $3<\pi$.

4.
$$\begin{aligned}1.53\overline{42}=1.53+0.00\overline{42}&=1.53+\sum_{n=1}^{\infty}0.0042(\frac{1}{100})^{n-1}\\&=\frac{153}{100}+\frac{42}{10000}\cdot\frac{1}{1-\frac{1}{100}}\\&=\frac{153}{100}+\frac{42}{10000}\cdot\frac{100}{99}\\&=\frac{153\cdot 99+42}{100\cdot 99}\\&=\frac{5063}{3300}\end{aligned}$$

5. For what value of x does this converge, and to what
$$\sum_{n=0}^{\infty}\frac{(x-2)^n}{3^n}=\sum_{n=0}^{\infty}(\frac{x-2}{3})^n$$
We know that this converges iff $|\frac{x-2}{3}|<1$ iff $|x-2|<3$ iff $-1<x<5$.

When it converges, it converges to $\frac{1}{1-\frac{x-2}{3}}=\frac{3}{5-x}$.

6. Using integral test, since the function is positive, decreasing and continuous
$$\sum_{n=0}^{\infty}\frac{1}{1+2n}\approx\int_1^{\infty}\frac{1}{1+2x}dx=\lim_{t\to\infty}\left[\frac{1}{2}\ln|1+2x|\right]_0^t=\infty$$
so it diverges.

Alternatively, using comparison test
$$\sum_{n=0}^{\infty}\frac{1}{1+2n}=1+\sum_{n=1}^{\infty}\frac{1}{1+2n}\geq 1+\sum_{n=1}^{\infty}\frac{1}{3n}=1+\frac{1}{3}\sum_{n=1}^{\infty}\frac{1}{n}$$
diverges.

7. Using integral test, since the function is positive, decreasing and continuous

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^3} \approx \int_1^{\infty} \frac{\ln x}{x^3} dx = \lim_{t\to\infty} \int_1^t \frac{\ln x}{x^3} dx = \lim_{t\to\infty} \left[\frac{\ln x}{-3x^2}\right]_1^t - \int_1^t \frac{1}{-2x^3} dx = \lim_{t\to\infty} \left[\frac{\ln x}{-2x^2} - \frac{1}{4x^2}\right]_1^t = \frac{1}{4}$$

by integration by parts $u = \ln x$, $du = \frac{1}{x}$, $v = \frac{1}{-2x^2}$, $dv = \frac{1}{x^3}$

Alternatively, using comparison test

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^3} = \sum_{n=1}^{\infty} \frac{\ln n}{n^3} \leq \sum_{n=1}^{\infty} \frac{1}{n^2}$$

since $\ln n < n$ for all $n \geq 1$ (because $\frac{d}{dx} \ln x = \frac{1}{x}$ so $\frac{d}{dx} \ln x \leq 1$ for all $x \geq 1$ and $\ln 1 = 0 < 1$). So it converges by p-test.

8. Using integral test, since the function is positive, decreasing and continuous

$$\begin{aligned}\sum_{n=1}^{\infty} \frac{1}{n^2+n^3} \approx \int_1^{\infty} \frac{1}{x^2+x^3} dx &= \int_1^{\infty} \frac{1}{x^2(x+1)} dx \\ &= \int_1^{\infty} -\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x+1} dx \\ &= \lim_{t\to\infty} \left[-\ln|x| + \frac{1}{-x} + \ln|x+1|\right]_1^t \\ &= \lim_{t\to\infty} \left[\ln|\frac{x+1}{x}| + \frac{1}{-x}\right]_1^t = \frac{1}{2}\end{aligned}$$

Alternatively, using comparison test

$$\sum_{n=1}^{\infty} \frac{1}{n^2+n^3} \leq \sum_{n=1}^{\infty} \frac{1}{n^2+n^2} = 2\sum_{n=1}^{\infty} \frac{1}{n^2}$$

converges by p-test.