Lecture 8

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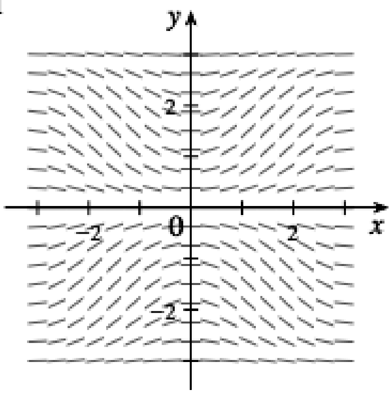
1. Sketch the graphs of the solutions that satisfy the given initial conditions:
   (a) $y(0) = 1$ (b) $y(0) = 2$ (c) $y(0) = -1$

   6. $y' = \sin x \sin y$

   II



2. Use Euler's method with step size 0.1 to estimate $y(0.5)$, where $y(x)$ is the solution of the initial-value problem $y' = y + xy$, $y(0) = 1$.

x	y	y'
0	1	1
0.1	1.1	1.21
0.2	1.221	1.465
0.3	1.368	1.778
0.4	1.545	2.163
0.5	1.762	

   so $y(0.5) \approx 1.762$.

3. $P(t)$ measures the performance of someone learning a skill after a training time t, M is the maximum level of performance, and k is a positive constant.

$$\begin{aligned} \frac{dP}{dt} &= k(M-P) \\ \int \frac{dP}{M-P} &= \int k dt \\ -\ln|M-P| &= kt + C \\ |M-P| &= e^{-kt-C} \\ P &= M - Ke^{-kt} \end{aligned}$$

   by letting $K = \pm e^{-C}$. Note that if we want $P(0) = 0$, then $K = M$ so $P(t) = M - Me^{-kt}$. The limit of this expression is $\lim_{t\to\infty} P(t) = M$.

4. A glucose solution is adminstered intravenously into the bloodstream at a constant rate r. As the glucose is added, it is convereted into other substances and removed from the bloodstream at a rate

that is proportional to the concentration at that time. Concentration at time $t = 0$ is C_0.

$$\frac{dC}{dt} = r - kC$$
$$\int \frac{dC}{r - kC} = \int dt$$
$$-\frac{1}{k}\ln|r - kC| = t + D$$
$$r - kC = Ke^{-kt}$$
$$C = \frac{1}{k}(r - Ke^{-kt})$$

so $C(0) = \frac{1}{k}(r - K) = C_0$ so $K = r - kC_0$. Thus $C(t) = \frac{1}{k}(r - (r - kC_0)e^{-kt})$ so

$$C(t) = (C_0 - \frac{r}{k})e^{-kt} + \frac{r}{k}$$

Assuming $C_0 < \frac{r}{k}$, $\lim_{t\to\infty} C(t) = \frac{r}{k}$ is the equalibrium amount of glucose in the blood, where it is adminstered and consumed at the same rate.

5. This is 7.7 #32 which was in your homework. The solution is

$$\int_0^{1.6} g(x)dx \approx S_8 = \frac{0.2}{3}[g(0)+4\cdot g(0.2)+2\cdot g(0.4)+4\cdot g(0.6)+2\cdot g(0.8)+\cdots+2\cdot g(1.2)+4\cdot g(1.4)+g(1.6)] = 19.21$$

The error is given by K being an upper bound to $|g^{(4)}(x)| \leq K$ for $0 \leq x \leq 1.6$, so the tightest bound is $K = 5$, since $-5 \leq g^{(4)}(x) \leq 2$. Thus the error is

$$\frac{5(1.6)^5}{180 \cdot 8^4} \approx .000071$$

6.
$$\int_0^2 z^2 \ln z dz = \lim_{t\to 0^+} \int_t^2 z^2 \ln z dz$$

now find the integral using integration by parts, with $u = \frac{z^3}{3}, du = z^2, v = \ln z, dv = \frac{1}{z}$

$$\int z^2 \ln z dz = \frac{z^3}{3}\ln z - \int \frac{z^3}{3}\frac{1}{z}dz = \frac{z^3}{3}\ln z - \frac{z^3}{9} + C$$

so
$$\int_0^2 z^2 \ln z dz = \lim_{t\to 0^+} \int_t^2 z^2 \ln z dz = \lim_{t\to 0^+} [\frac{z^3}{3}\ln z - \frac{z^3}{9}]_t^2 = \frac{8}{3}\ln 2 - \frac{8}{9} - \frac{t^3}{3}\ln t + \frac{t^3}{9}$$

using L'Hoptial's rule

$$\lim_{t\to 0^+} t^3 \ln t = \lim_{t\to 0^+} \frac{\ln t}{1/t^3} = \lim_{t\to 0^+} \frac{1/t}{-3/t^4} = \lim_{t\to 0^+} -\frac{1}{3}t^3 = 0$$

thus the answer is
$$\int_0^2 z^2 \ln z dz = \frac{8}{3}\ln 2 - \frac{8}{9}$$