Lecture 4

Enoch Cheung

January 23, 2014

1.
$$\int \frac{x^2+1}{(x-3)(x-2)^2}dx = \int \frac{10}{x-3} + \frac{-9}{x-2} + \frac{-5}{(x-2)^2} = 10\ln|x-3| - 9\ln|x-2| + \frac{5}{x-2} + C$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$x^2+1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3) = (A+B)x^2 + (-4A-5B+C)x + (4x+6B-3C)$
thus $A = 10, B = -9, C = -5$.

2.
$$\int \frac{x^3+4}{x^2+4}dx = \int x + \frac{-4x+4}{x^2+4}dx = \int x + \frac{-4x}{x^2+4} + \frac{4}{x^2+4}dx = \frac{x^2}{2} - 2\ln|x^2+4| + 2\tan^{-1}(\frac{x}{2}) + C$$

using long division.

3.
$$\int_0^1 \frac{x^3+2x}{x^4+4x^2+3}dx = \int_0^1 \frac{x^3+2x}{(x^2+3)(x^2+1)}dx = \frac{1}{2}\int_0^1 \frac{u+2}{(u+3)(u+1)}du$$
$$= \frac{1}{2}\frac{1/2}{u+3} + \frac{1/2}{u+1}du = \frac{1}{4}(\ln|u+3| + \ln|u+1|)\Big]_0^1 = \frac{1}{4}(\ln 4 + \ln 2 - \ln 3 - \ln 1) = \frac{1}{4}\ln\frac{8}{3}$$

using $u = x^2$, so $du = 2xdx$, note that $1^2 = 1$ and $0^2 = 0$.

$$\frac{u+2}{(u+3)(u+1)} = \frac{A}{u+3} + \frac{B}{u+1}$$
$$u+2 = A(u+1) + B(u+3)$$

so $A + B = 1$ and $A + 3B = 2$ so $A, B = \frac{1}{2}$.

4.
$$\int \frac{dx}{x(x^2+4)^2} = \int \frac{du}{2x^2(x^2+4)^2} = \frac{1}{2}\int \frac{du}{u(u+4)^2} = \frac{1}{2}\int \frac{1/16}{u} + \frac{-1/16}{u+4} + \frac{-1/4}{(u+4)^2}$$
$$= \frac{1}{2}(\frac{1}{16}\ln|u| + \frac{-1}{16}\ln|u+4| + \frac{1}{4}\frac{1}{u+4}) + C = \frac{1}{32}\ln|x^2| - \frac{1}{32}\ln|x^2+4| + \frac{1}{8}\frac{1}{x^2+4} + C$$

using $u = x^2$ so $du = 2xdx$.

$$\frac{1}{u(u+4)^2} = \frac{A}{u} + \frac{B}{u+4} + \frac{C}{(u+4)^2}$$
$$1 = A(u+4)^2 + Bu(u+4) + Cu = (A+B)u^2 + (8A+4B+C)u + 16A$$

so $A = 1/16, B = -1/16, C = -1/4$.