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# Modeling Intratumor Gene Copy Number Heterogeneity using <br> Fluorescence in Situ Hybridization data 

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## Tumor heterogeneity


(2) Copy numbers for a single gene
(D)
(1)
(E)
(o)

## Tumor heterogeneity

- Inverse problem approach
- High-throughput DNA sequencing data by Oesper, Mahmoody, Raphael (Genome Biology 2013)
- SNP array data by Van Loo et al. (PNAS 2010), Carter et al. (Nature Biotechnology 2012)


## Tumor heterogeneity



FISH data, direct assessment


## FISH data

## Multidimensional histogram on the

 positive integer cone, e.g., for 2 dimensions

## FISH data

- Let $x_{i j}$ be the number of copies of gene $j$ in the $i$-th cell, where $\mathrm{i}=1, . ., \mathrm{n}(\sim 100)$ and $\mathrm{j}=1, . ., \mathrm{g}(\sim 10)$.
- The bounding box's size
$\left|\left[\min _{i} x_{i 1}, \max _{i} x_{i 1}\right] \times . . \times\left[\min _{i} x_{i g}, \max _{i} x_{i g}\right]\right|$ typically grows exponentially in the number of probes for the breast cancer datasets
- This feature seems to be tumor dependent, i.e., does not hold necessarily for all cancers


## FISH data

- Breast and cervical cancer data publicly available from NIH


## ftp://ftp.ncbi.nlm.nih.gov/pub/FISHtrees/data

## Motivation

- Understanding tumor heterogeneity is a key step towards:
- find first mutation events, hence identify new drugs and diagnostics
- predict response to selective pressure, hence develop strategies to avoid drug resistance
- identify tumors likely to progress, hence avoid over- and under-treatment.


## Related work

- Pennington, Smith, Shackney and Schwartz (J. of Bioinf. and Comp. B. 2007)
- Two probes
- Random walk where coordinate $i$ is picked independently and with probabilities pio, $\mathrm{pi}-1, \mathrm{pii}$ is modified by $\{0,-1,+1\}$ respectively.
- Efficient heuristic to maximize a likelihood function over all possible trees and parameters.


## Related work

- Chowdhury, Shackney, Heselmeyer-Haddad, Ried, Schäffer, Schwartz (Best paper in ISMB'13). Among other contributions:
- Methods which are able to handle large number of cells and probes.
- Exponential-time exact algorithm and an efficient heuristic for optimizing their objective
- New test statistics, tumor classification
- Extensive experimental evaluation


## Related work

Copies of Gene 2


Copies of gene 1

Chowdhury et al.:

- Problem: Find tree (and possibly Steiner nodes) to minimize cost of connecting all input (terminal) vertices


## Contributions I

- Probabilistic approach
- We summarize the empirical distribution based on a model that captures complex dependencies among probes without over-fitting.
- Allows us to assign weights on the edges of the positive integer di-grid which capture how likely a transition is.
- And now, how do we derive a tumor phylogeny?..


## Proposed method

- Let $X_{j}=$ \#copies of gene j
- integer valued random variable
- Let $I_{j}$ be the domain of $X_{j}$
- We model the joint probability distribution $X=\left(X_{1}, . ., X_{g}\right)$ as

$$
\operatorname{Pr}(x)=\frac{1}{Z} \prod_{A \subseteq[g]} e^{\varphi_{A}(x)} \searrow_{\downarrow}
$$

$$
x=\left(x_{1}, \ldots, x_{g}\right)
$$

Potential function

## Proposed method

- with the following properties of hierarchical log-linear model
- log-linearity: the logarithm of each potential depends linearly on the parameters, e.g., for $\mathrm{g}=2, \mathrm{I}_{1}=\mathrm{I}_{2}=\{0,1\}$ then,

$$
\begin{aligned}
\log \operatorname{Pr}[x] & =w_{0}+w_{(1) 0} \mathbb{1}\left\{x_{1}=0\right\}+w_{(1) 1} \mathbb{1}\left\{x_{1}=1\right\}+w_{(2) 0} \mathbb{1}\left\{x_{2}=0\right\} \\
& +w_{(2) 1} \mathbb{1}\left\{x_{2}=1\right\}+w_{(12) 00} \mathbb{\mathbb { 1 }}\left\{x_{1}=0, x_{2}=0\right\}+w_{(12) 01} \mathbb{1}\left\{x_{1}=0, x_{2}=1\right\} \\
& +w_{(12) 10} \mathbb{\mathbb { 1 }}\left\{x_{1}=1, x_{2}=0\right\}+w_{(12) 11} \mathbb{\mathbb { 1 }}\left\{x_{1}=1, x_{2}=1\right\},
\end{aligned}
$$

## Proposed method

- Hierarchical:
- $A \subseteq B, w_{A}=0 \rightarrow w_{B}=0$
- For instance $w_{\{1,2,3\}}$ can be non-zero only if $\mathrm{w}_{\{1,2\}}, \mathrm{w}_{\{1,3\}}, \mathrm{w}_{\{2,3\}}$ are non-zero.
- Allows significant computational savings compared to the general form
- Biologically meaningful: if a set A of genes does not interact, any superset of A maintains this property.


## Proposed method

- A lot of related work and off-the-shelf software for learning the parameters
- Based on Zhao, Rocha andYu who provide a general framework that allows us to respect the 'hierarchical' property ..
- ... Schmidt and Murphy provide efficient optimization algorithms for learning a hierarchical log-linear model


## Proposed method

- We use the learned hierarchical log-linear model in two ways
- The non-zero weights provide us insights into dependencies of factors
- We use them to assign weights on the positive integer di-grid


## Proposed method

Copies of Gene 2


Copies of gene 1


Nicholas Metropolis Given a probability distribution $\boldsymbol{\pi}$ on a state space we can define a Markov Chain whose stationary distribution is $\pi$.

## Contributions II

- Question: Can we use the wealth of
 inter-tumor phylogenetic methods to understand intra-tumor cancer heterogeneity?


## Contributions II

- Motivated by this question:
- We prove necessary and sufficient conditions for the reconstruction of oncogenetic trees, a popular method for inter-tumor cancer inference
- We exploit these to preprocess a FISH dataset into an inter-tumor cancer dataset that respects specific biological characteristics of the evolutionary process


## Oncogenetic Trees

- Desper, Jiang, Kallioniemi, Moch, Papadimitriou, Schäffer
- T(V,E,r) rooted branching
- $\mathrm{F}=\left\{\mathrm{Al}_{1}, ., \mathrm{Am}\right\}$ where $\mathrm{A}_{\mathrm{i}}$ is the set of vertices of a rooted sub-branching ofT.
- What are the properties that F should have in order to uniquely reconstruct $T$ ?
- Let T be consistent with F if it could give rise to F.


## Example

Onco-tree


Patient $1, A_{1}=\{r, a, b, c\}$

Patient $2, A 2=\{r, a, b\}$

Patient $3, A 3=\{r, a, b, d\}$

Also, consistent with $\left\{A_{1}, A_{2}, A_{3}\right\}$




## Oncogenetic Trees

- Theorem
- The necessary and sufficient conditions to reconstruct $T$ from $F$ are the following:
- $x, y$ such that $(x, y)$ is an edge, there exists a set in the family that contains $x$ but not $y$.



## Oncogenetic Trees

- If $x$ is not a descedant of $y$ and vice versa then there exist two sets $\mathrm{Ai}_{1} \mathrm{Aj}_{\mathrm{j}}$ such that
- $x$ is in $A_{i}$ but not in $A_{j}$
- $y$ is in $A_{j}$ but not in $A_{i}$
necessity



## Oncogenetic trees

- It turns out that the necessary conditions are sufficient (constructive proof)
- Allows us to force an oncogenetic tree to capture certain aspects of intratumor heterogeneity dynamics


## Contributions III

- We evaluate our method on real FISH data where we show findings consistent with cancer literature
- Here, we show results for a breast cancer dataset


## Experimental results

- No ground truth, but
- concurrent loss of cdhı function and p53 inactivation play a key role in breast cancer evolution
- subsequent changes in ccndı, myc, znf217 according to our tree are consistent with oncogenetic literature



## Conclusions

- There exists a lot of interest in understanding intra-tumor heterogeneity
- Releasement of FISH data that assess it directly can promote this understanding
- Concerning our work:
- Better algorithms for fitting the model
- Allow higher-order interactions but use additional penalty (e.g., AIC)


## Conclusions

- ... concerning our work
- Other choices of inter-tumor methods
- Tumor classification applications
- Consensus FISH trees
- Allow more mechanisms in copy number changes
- Understand better the connection between our work and Chowdhury et al.


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## Thanks!

Appendix

## Experimental results



## Experimental results

## Generated with code available at ftp://ftp.ncbi.nlm.nih.gov/pub/FISHtree



