21-111 Calculus I - Fall 2004

First Course Test

September 27, 2004

Name:

Recitation Group:

There are 5 problems on this exam. Complete all problems, showing all work. Extra space is given on page 7. Please indicate clearly if you use the extra space. Good luck.

Problem	Points	Score
1	20	
2	15	
3	20	
4	20	
5	25	
Total	100	

1. [16 points] Given

$$f(x) = x + \frac{1}{x} \text{ for } x \geq 1$$
$$g(x) = 1 + x^2 \text{ for } -\infty < x < \infty$$

find the functions
$$(f(g(x))), (g \circ f), (f(f(x)))$$

if they exist. Make sure to specify the domains of the functions.

Solution: $f(g(x)) = 1 + x^2 + \frac{1}{1+x^2}$ with domain x any real number (as $1 + x^2 \geq 1$ for all x).

$g(f(x)) = 3 + x^2 + \frac{1}{x^2}$ with domain $x \geq 1$.

$f(f(x)) = \frac{(x^2+1)^2 + x^2}{x(x^2+1)}$ with domain $x \geq 1$.

[4 points] Evaluate $(f(g(t^2)))$ and $[(f(g(2)))]^2$.

Solution: $f(g(t^2)) = 1 + t^4 + \frac{1}{1+t^4} = \frac{2+2t^4+t^8}{1+t^4}$

$[(f(g(2)))]^2 = 27\frac{1}{25} = \frac{26^2}{25}$

2. [15 points] What is the value after 4 years of $ 600 invested at 6% annual interest compunded twice a year? [Hint: $(1.03)^8 \approx 1.27$.]

Solution: $600(1 + \frac{0.06}{2})^{4 \cdot 2} = (\$)762$

3. A factory produces $N(t) = 3t - 3$ robot arms after t hours of operation. The fixed cost of production is $ 5000 and the cost to produce one robot arm is $ 700. The revenue for selling x robot arms is $R(x) = 800x - 200$.

   (a) [8 points] How many arms must the company make and sell for the revenue to equal the cost?
   Solution: $R(x) = C(x)$ when $x = 52$.

   (b) [10 points] Find a function representing the profit made by operating the factory for t hours (Assume all the arms manufactured are also sold.) **Solution:** $P(t) = 300t - 5500$
   [2 points] Find the initial profit or loss ($t = 0$). At what rate does the profit increase in each hour? **Solution:** Initial loss $P(0) = -5500$ and increase per hour $ 300.

4. A corporation builds printers. After operating for t months, their factory in Bangkok produces

$$B(t) = 300t$$

printers. In the same amount of time, their factory in Madrid produces

$$M(t) = 100t + 10t^2$$

printers. The revenue generated from selling x printers is $R(x) = 200x - 300$ dollars.

(a) [10 points] After how many months of operation have the factories produced the same number of printers? **Solution:** $B(t) = M(t)$ when $t = 0$ or $t = 20$.

(b) [10 points] What is the revenue generated for the corporation by these factories after t months of operation? **Solution:** $R(B(t) + M(t)) = 80,000t + 2000t^2 - 300$

5. Simplify the following, leaving only one fraction and no negative exponents.

(a) [8 points] $\dfrac{\frac{xy}{(x-y)}}{\frac{x^3}{y} \cdot \frac{y^3}{x}}$ **Solution:** $\dfrac{1}{(x-y)xy}$

(b) [10 points] $\dfrac{\frac{x}{y} - \frac{1}{x}}{\frac{y}{2x} + \frac{x}{2y}} - \dfrac{x-y}{xyz}$ **Solution:** $\dfrac{2(x^2-y)xyz - (x-y)(x^2+y^2)}{(x^2+y^2)xyz}$

(c) [7 points] $\left(\dfrac{4x^{-8}}{9y^6}\right)^{-\frac{1}{2}}$] **Solution:** $\frac{3}{2}y^3x^4$

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