21-111 Calculus I - Fall 2004

Review for Final

December 1, 2004

The questions on the final will be close to the questions on the review. The Final will cover the whole course. There will be no question on the algebraic review but all the topics covered in the first two weeks of class will be needed to answer the other questions on the exam.

1. Find the equation of the line L passing through $(-1,-4)$ and $(2,5)$ in slope intercept form. Find the lines L_1 parallel to L and L_2 perpendicular to L passing through $(2,1)$.
2. Let $f(x)=\frac{x^2-1}{x+2}$, $g(x)=\sqrt{x-1}$, and $h(x)=x^2-1$. Find the following functions and their domains:
   (a) $f(x)+g(x)\cdot h(x)$
   (b) $f(g(x))$
   (c) $g(h(x))$
3. Find the following limits:
   (a) $\lim\limits_{h\to 0}\dfrac{\frac{1}{x-2-h}-\frac{1}{x-2}}{h}$
   (b) $\lim\limits_{h\to 0}\dfrac{\sqrt{3+h}-\sqrt{3}}{h}$
   (c) $\lim_{x\to 3^-}\frac{x^2-2}{x-3}$
   (d) $\lim\limits_{x\to -4}\dfrac{\frac{1}{4}+\frac{1}{x}}{4+x}$

(e) $\lim_{x \to 9} \dfrac{9 - x}{3 - \sqrt{x}}$

4. A ball is given a push so that it has an initial velocity of 5 m/s down an inclined plane, than the distance it has rolled after t seconds is $s = 5t + 3t^2$.

   (a) Find the velocity after 2 s.

   (b) How long does it take to reach the velocity of 35 m/s?

   (c) What is the acceleration at time t?

5. Let $C(x)$ be the cost (in \$) of manufacturing x bicycles per day in a certain factory. The Cost of producing 50 bikes is 5000 and the marginal cost of 50 bikes is 45.

   (a) Estimate the cost of producing 52 bikes. (Assume that the Cost function is not linear)

   (b) Let $C(x) = 3x^2 - 245x + 9750$. What is the actual cost of producing 52 bikes and what is the marginal cost at 52 bikes?

6. An initial amount of \$3000 is placed into an account for four years at an interest rate of 6%.

   (a) What is the amount in the account after four years if the interest is compounded 3 times a year?

   (b) What is the amount in the account after four years if the interest is compounded continuously?

7. Let

$$f(x) = \begin{cases} \frac{1}{x+1} & \text{if} \quad x \leq 0 \\ 3x + 1 & \text{if} \quad x > 0 \end{cases}$$

   Does $f(x)$ have any discontinuities? If yes, where are they? Where is $f(x)$ differentiable?

8. Let $f(x) = x^4 - 2x^3 - 3x^2 + 8x - 4$.

   (a) Find the x- and y-intercepts of $f(x)$.

   (b) Find the relative maxima and minima of the function.

(c) Find the intervals where the function is increasing and decreasing

(d) Where are the points of inflection?

(e) Find the intervals where the function is concave up and where it is concave down.

9. Sketch the following graphs. To do that find their x- and y-intercepts, relative maxima/minima, points of inflection and asymptotes if they exist:

   (a) $f(x) = x^4 - 4x^3 + 2x^2 + 4x - 3$

   (b) $g(x) = \frac{x^2+2*x+1}{x-1}$

10. A rectangular plot of farmland is bounded on one side by a river and on the three other sides by a single stranded electrical fence. With 800 m of fence at our disposal, what is the largest area that can be enclosed?

11. A manufacturer has been selling 1000 television sets a week at \$450 each. A market survey indicates that for each rebate of \$10 offered to the buyer, the number of sets sold will increase by 100 per week.

    (a) Find the demand function.

    (b) How large a rebate should the company offer the buyer in order to maximize its revenue?

    (c) If its weekly cost function is $C(x) = 68,000 + 150x$ how should the manufacturer set the size of the rebate in order to maximize its profit?

12. Differentiate the following functions (use the rules given in brackets when applicable):

    (a) $f(x) = \sqrt{x+1}$ (limit of differential quotient)

    (b) $f(x) = \frac{(x^3-2x+1)^2}{3x^2+2x+1}$ (quotient rule)

    (c) $f(x) = \frac{(x^2+1)^{\frac{1}{2}}+1}{x^2+1}$ (chain-rule and quotient rule)

    (d) $f(x) = (x^2+2x+3)^{\frac{3}{2}}(x+1)(x^3+2x+1)^{\frac{1}{2}}$ (logarithm rule)

    (e) $f(x) = e^{(x+1)^2}(x^2)$

    (f) $f(x) = ln(x^3+2x^2+4)$

(g) $f(x) = x^2 - 3ln(\frac{x^2-1}{x^3+1}) - e^{x^2}$

13. Find the equation for the circle with center at $(1, 1)$ and radius $r = 2$. What is the slope of the circle at the point $(0, \sqrt{3} - 1)$.

14. What is the slope of $(x^2 - 1)y^2 + 3y = 3xy$ at $(1, 1)$?

15. Two truck convoys leave a depot, convoy A is traveling east at 40 mph and convoy B is traveling north at 30 mph. How fast is the distance between the convoys changing 6 min later, when convoy A is 4 miles from the depot and convoy B is 3 miles from the depot?

16. A spherical balloon is filled with helium at a rate of $100\pi ft^3/min$. How fast is the balloon's radius increasing at the instant when the radius is 5ft? How fast is the surface area increasing at that point? [Hint: The first number given is the change in Volume of the balloon. The volume of a sphere is $V = \frac{4}{3}\pi r^3$ and the surface area is $A = 4\pi r^2$.]

17. Let $f(x) = e^{\frac{3}{2}x}e^{\frac{7}{4}x}$ and $g(x) = 3ln(x+1)+2ln(x)-\frac{1}{2}ln(x)$. Simplify the functions as far as possible (get functions of the form $e^k x$ and $ln(h(x))$). Find $f(g(x))$.

18. At the beginning of 1988, 9.9 mio. people were living in Beijing and the population was growing exponentially. In 1992 the population had grown to 11.0 mio. Assuming that the growth is continuous, what was the estimated population in 2000?

19. The charcoal from a tree killed in the volcanic eruption that formed Crater Lake in Oregon contained 44.5% of the carbon-14 found in living wood. Estimate the age of Crater Lake. [Hint: Remember that the decay-constant of Carbon-14 is.00012]

20. Suppose that the price of wheat per bushel at time t (n months) is approximated by $f(t) = 4 + .001t + .01e^{-t}$. What is the percentage rate of change of $f(t)$ at t=0 and t=2?

21. An electronics store can sell $q = 10,000/(p + 50) - 30$ cellular phones at a price p dollars per phone. The current price is \$150.

    (a) Is demand elastic or inelastic at $p = 150$?

    (b) If the price is slightly lower, will revenue increase or decrease?