21-111 Calculus I - Fall 2004

Review 3

November 12, 2004

1. A cylindrical can without top but with a bottom is made from $300\pi in^2$ of sheet metal. No sheet metal will be wasted. What is the greatest Volume of such a can? [Hint: Volume of a can=Area of circle * height of can]

2. A baseball team plays in a stadium that holds 55000 spectators. With ticket prizes at $ 10, the average attendance had been 27000. When ticket prizes were lowered to $ 8, the average attendance rise to 33000.

   (a) Find the demand function assuming that it is linear.

   (b) How should ticket prizes be set to maximize revenue?

3. A manufacturer has been selling 1000 television sets a week at $ 450 each. A market survey indicates that for each rebate of $ 10 offered to the buyer, the number of sets sold will increase by 100 per week.

   (a) Find the demand function.

   (b) How large a rebate should the company offer to maximize its revenue?

   (c) If the weekly cost function is $C(x) = 68,000 + 150x$, how should the manufacturer set the size of the rebate in order to maximize profit?

4. A supermarket manager estimates that a total of 800 cases of soup will be sold at a steady rate during the coming year and it costs $ 4 to store a case for a year. The average number of cases stored is $\frac{1}{2}x$ where x is the number cases per order placed. The cost per order will be $ 100. What is the optimal reorder quantity that minimizes cost?

5. Differentiate the following functions:

   (a) $f(x) = (x^2 + 3x - 1)^3 (x^3 - 1)^{\frac{1}{2}}$

   (b) $g(x) = \dfrac{3x^2 - x^{\frac{3}{2}}}{3x^2 - 1}$

   (c) $h(x) = \dfrac{\sqrt[3]{x^4 - 3x^2}}{\sqrt{x - 1}}$

6. Find two functions $g(x)$ and $h(x)$ such that the functions $f(x)$ below are $f(x) = g(h(x))$ and differentiate $f(x)$ using the chain rule:

   (a) $f(x) = \sqrt{\frac{z-1}{z+1}}$

   (b) $f(x) = (x^2 + 3x - 4)^{\frac{3}{4}}$

7. Differentiate $f(x) = (1 + 4x)^5 (3 + x - x^2)^8$

8. Use implicit differentiation to find the slope of the equation $y^4 + x^2y^2 + x^4 = y + 1$.

9. What is the slope of the equation $x^2 + 2xy - y^2 + x = 2$ at the point $(1, 2)$?

10. A baseball diamond is a square with 90 ft sides. A batter hits the ball and runs toward first base with a speed of 24 ft/s. [Hint: Think of the baseball diamond as a square with corners A, B, C and D. After hitting the ball, the runner runs from point A to point B (first base)]

    (a) At what rate is his distance from second base (point C) decreasing when he is halfway to first base? [Hint: Draw a picture and then use Pythagoras theorem and related rates for both parts of the problem.]

    (b) At what rate is his distance from third base (point D) pdflatexincreasing at the same moment?

11. A plane is flying horizontally at an altitude of 1 mile and at a speed of 500 mi/h. Find the rate at which the distance of the plane to the radar station is increasing when it is 2 miles away from the station.

12. Write each of the functions as e^{kx}:

(a) $(\sqrt[3]{e^{-x}e^{5x}})^3$

(b) $\dfrac{e^{-\frac{1}{2}}e^{4x}}{e^{2x}e^{-\frac{3}{4}}}$

13. Differentiate the following functions:

(a) e^{3x^2-5x+2}

(b) $(1+2x^3)e^{3x}$

(c) $\sqrt{e^{-\frac{1}{2}x}+3x^4}$