

21-111 Calculus I - Fall 2004

Review 2

October 18, 2004

1. Consider a rectangular box with square base and height $=\frac{1}{2}$ length. Assume that the box has no cover and that it is made of a material costing \$4 per square-inch. Write an equation taking into consideration that the total cost for the box is \$ 1200. Find the length of the box.
2. Find the equation of the line L through points $(3, 2)$ and $(4, -1)$. Find the equations of the lines L_1 perpendicular and L_2 parallel to L passing through $(1, 1)$. Draw the graphs of the lines.
3. Assume that a machine is worth \$25,000 one year after its purchase and \$20,000 three years after its purchase. (Assume the machine loses the same amount in value every year).
 - (a) Find the formula for the depreciation of the machine in terms of t years.
 - (b) What was the machine worth when it was bought?
 - (c) When is the machine worth \$0?
 - (d) How much is the machine worth after 8 years?
4. Find the derivative of $f(x)$ using the secant-line calculation if
 - (a) $f(x) = \sqrt{x+1}$
 - (b) $f(x) = \frac{1}{x^2}$
5. Find the derivative of $f(x)$ at $x = 2$ using the difference quotient and limits if

- (a) $f(x) = x^3$
 (b) $f(x) = \frac{5}{x} - 1$

6. Find the following limits using the limit rules and theorems:

- (a) $\lim_{x \rightarrow 2} x^3 - 5x + 2$
 (b) $\lim_{x \rightarrow -2} \left(\frac{3x^2 - x - 14}{x^2 - 4} \right)$
 (c) $\lim_{x \rightarrow \infty} \left(\frac{2x^5 - 3x^2 + 13}{x^4 - 2x^2 + 3} \right)$
 (d) $\lim_{h \rightarrow 0} \left(\frac{\frac{1}{4-h} - \frac{1}{4}}{h} \right)$
 (e) $\lim_{h \rightarrow 0} \left(\frac{\sqrt{(1+h)^2 + (1+h)} - 1 - 1}{h} \right)$

7. Find where the following functions are continuous and where they are differentiable:

(a)

$$f(x) = \begin{cases} 2x^2 - 1 & \text{for } x \leq -1 \\ x + 2 & \text{for } -1 < x < 1 \\ \frac{1}{2}x^2 + \frac{5}{2} & \text{for } x \geq 1 \end{cases}$$

(b)

$$f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{for } x < 1 \\ 3x - 1 & \text{for } x > 1 \end{cases}$$

8. Let $y = 3x^{-\frac{1}{2}} + x^4$. Find $\frac{d^2y}{dx^2}$ and $\frac{d^2y}{dx^2} \Big|_{x=2}$.

9. An arrow is shot upwards on the moon with a velocity of 58 meters per second. Its height after t seconds is given by $H(t) = 58t - 0.83t^2$.

- (a) Find the velocity of the arrow after one second.
 (b) Find the velocity of the arrow when $t = a$.
 (c) When will the arrow hit the moon?
 (d) With what velocity will the arrow hit the moon?

- (e) What is the acceleration or deceleration of the arrow after 2 seconds and after 20 seconds?
10. The cost (in \$) of producing x units of a certain commodity is given by a function $C(x)$.
- (a) If $C(100) = 6500$ and $C'(100) = 20$, approximate the cost of producing 105 units.
- (b) Assume that $C(x) = 5000 + 10x + 0.05x^2$. Find the average rate of change of C with respect to x when the production level is changed from $x = 100$ to $x = 105$.
- (c) Find the marginal cost when $x = 105$.
11. Let $f(x) = -x^2 + 2x$ and $g(x) = -x^3 + 6x^2 - 12x + 8$, both defined on $0 \leq x \leq 3$. Find the zeros, local and absolute maxima and minima of the two graphs, the points where the graphs intersect and possible points of inflection of the two graphs. Use this information to draw the two graphs (in one coordinate system).