Department of Mathematics
Carnegie Mellon University

21-256 Multivariate Analysis and Approxim
October 24, 2005

Review Problems for Test 2

1. Find the parametric equations for the line passing through the points $(-1, 2, 5)$ and $(3, -4, 2)$.
2. Find the symmetric equations for the line that passes through the point $(-1, 0, 4)$ and is parallel to the line given by
$$x = t \quad y = -1 + 3t \quad z = 4 - t.$$
3. Show that the line that passes through the points $(3, 2, -1)$ and $(-2, 1, -5)$ is perpendicular to the line that goes through the points $(2, -3, 0)$ and $(3, -4, -1)$.
4. Find the symmetric equations for the line that passes through $(1, -1, 3)$ and is perpendicular to the plane $3x + y - z = 10$. In addition, find the point of intersection of the line with the plane.
5. Find the equation of the plane containing the lines
$$L_1 : \qquad x = 2t - 1 \quad y = -3t \quad z = -t - 4$$
$$L_2 : \qquad x = 4s + 3 \quad y = s + 1 \quad z = 2s - 2.$$
6. Find the angle between the planes $2x + y - z = 10$ and $x - y + 3z = 2$.
7. Find the following limits if they exist, or show that they do not exist.
   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x + y}{x - y}$
   (b) $\displaystyle\lim_{(x,y)\to(-1,1)} (x^2 y - 3x^3 y^2 + 4)$
   (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x - y}$
8. Show that the function
$$f(x, y) = \begin{cases} \frac{x^2y^2+y^4}{x^4+y^4} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$
is discontinuous at $(0, 0)$.
9. Find the second partial derivatives of
$$f(x, y) = e^{(x^2+y^2)}$$
10. In the following problems, find the indicated derivatives.
    (a) $w(s, t) = \ln(st), \quad s(x, y) = x + y, \quad t(x, y) = y/x, \quad w_x =?, \quad w_y =?$
    (b) $f(u, v) = u/v, \quad u(s) = s^2 + 1, \quad v(s) = e^{s-1}, \quad \dfrac{df}{ds} =?$

11. Find the maximum rate of change of $f(x, y, z) = \ln(xyz)$ at the point $(-1, 2, -2)$ and the direction in which the maximum occurs.

12. Classify the critical points of the following functions

    (a) $f(x, y) = e^x \sin(y)$

    (b) $f(x, y) = x^2 + y^2 + x^{-2}y^{-2}$

    (c) $f(x, y, z) = x^2 + y^2 + z^2 + xy$

13. Find the absolute maxima and minima of the function $f(x, y) = 2\sin x + 5\cos y$ over the rectangle with vertices $(0, 0)$, $(2, 0)$, $(2, 5)$, and $(0, 5)$.

14. Find the absolute maxima and minima of the function $f(x, y) = e^{x^2-y^2}$ over the disk $x^2 + y^2 \leq 1$.

15. A company manufactures and sells 2 products, snarfblatts and dinglehoppers, that sell for \$10 and \$9 per unit, respectively. The cost of producing x snarfblatts and y dinglehoppers is

$$C(x, y) = 400 + 2x + 3y + 0.01(3x^2 + xy + 3y^2).$$

    Find the values of x and y that maximize the companies profit (profit = revenue - cost).

16. A T-shirt shop carries two competing shirts, one endorsed by Noddy and the other by Big-Ears. The owner of the store can get both types at a cost of \$2 per shirt, and estimates that if Noddy shirts are sold for x dollars apiece and Big-Ears shirts for y dollars apiece, consumers will buy approximately $40 - 50x + 40y$ Noddy shirts and $20 + 60x - 70y$ Big-Ears shirts each day. How should the owner price the shirts in order to maximize the profit?

17. Use the method of Lagrange multipliers to solve the following problems.

    (a) Maximize $f(x, y) = x^2 - 2y - y^2$ subject to $x^2 + y^2 = 1$.

    (b) Minimize $f(x, y) = x^2 + y^2 + z^2$ subject to $4x^2 + 2y^2 + z^2 = 4$.

    (c) Maximize $f(x, y, z) = xyz$ subject to $x^2 + y^2 = 3$ and $y = 2z$.

18. A manufacturer is planning to sell bottles of frobscottle at the price of \$150 per bottle, and estimates that if x thousand dollars is spent on development and y thousand dollars on promotion, approximately

$$\frac{320y}{y+2} + \frac{160x}{x+4}$$

    bottles will be sold. The cost of manufacturing the frobscottle is \$50 per bottle. If the manufacturer has a total of \$8,000 to spend on development and promotion, how should this money be allocated to generate the largest possible profit?