Department of Mathematics
CARNEGIE MELLON UNIVERSITY
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21-257, Methods and Models for Optimization, Spring 1999

Review for Test 2

Test 2 will cover all of Chapter 3. To review for the test, please look at the homework problems and the similar ones in the book.

The test will have 5 problems, each one worth 20 points. The actual length of the problems on the test will be shorter than the homework problems - they will be "one-step" problems. The following list should give you an idea for the length and level of difficulty.

At the same time, you should keep in mind that the test will cover all the sections of Chapter 3, which means that you should expect topics not represented on this review sheet - for those just look at your homework problems.

Review Problems

1. Consider the following LP.

$$\begin{array}{rrcrcr} \text{Maximize:} & 40x_1 & + & 30x_2 & & \\ \text{Subject to:} & x_1 & + & x_2 & \leq & 55 \\ & 2x_1 & + & 3x_2 & \leq & 130 \\ & 60x_1 & + & 10x_2 & \leq & 1100 \\ & 10x_1 & + & 20x_2 & \leq & 900 \\ & x_1 \geq 0, x_2 \geq 0 & & & & \end{array}$$

   Insert slack variables and set up a simplex tableau (but do not solve the LP).

2. Consider the following tableau.

x_1	x_2	x_3	s_1	s_2	s_3	
1	2	0	0	$-\frac{1}{2}$	0	120
0	-3	1	1	0	0	55
0	5	2	0	5	1	104
0	2	-3	0	-1	2	1700

   Apply one step of the simplex algorithm to this tableau.

3. In the process of solving some LPs, the following tableaux were obtained.

   (a)

x_1	x_2	x_3	s_1	s_2	
1	3	0	1	2	16
0	-2	1	1	1	7
0	-3	0	-2	1	26

   (b)

x_1	x_2	s_1	s_2	s_3	
1	0	$-\frac{1}{2}$	0	1	10
0	1	-1	0	$\frac{3}{2}$	15
0	0	-5	1	0	25
0	0	-1	0	2	1020

(c)

x_1	x_2	x_3	s_1	s_2	s_3	
1	0	0	1	1	-1	11
0	0	1	$-\frac{1}{2}$	1	0	12
0	1	0	0	3	2	36
0	0	0	1	3	5	7090

For each of these, determine the situation that it describes, namely, one of the following:

- the optimal tableau has been reached;
- the problem has no upper bound;
- at least one more step of the simplex algorithm can be made.

Give reasons to support your answer.

4. Consider the following LP. Insert slack, surplus, and artificial variables and determine the artificial objective function for the two-phase method. Set up the corresponding tableau for the two-phase method.

$$\begin{array}{rrcrcrcr} \text{Maximize:} & 10x_1 & + & 40x_2 & + & 20x_3 & & \\ \text{Subject to:} & 60x_1 & + & 30x_2 & + & 10x_3 & \leq & 1200 \\ & x_1 & + & 2x_2 & & & \geq & 1 \\ & x_1 & + & x_2 & + & 3x_3 & = & 8 \end{array}$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

5. Consider the following minimizing problem.

$$\begin{array}{rrcrcrcr} \text{Minimize:} & 2y_1 & + & 5y_2 & + & 4y_3 & & \\ \text{Subject to:} & 3y_1 & + & 2y_2 & + & 5y_3 & \leq & 30 \\ & 2y_1 & + & 4y_2 & - & 3y_3 & \geq & 40 \\ & 2y_1 & + & 6y_2 & + & 4y_3 & = & 80 \end{array}$$
$$y_1 \geq 0, y2 \text{ unrestricted}, y_3 \geq 0$$

Formulate the dual maximizing problem.