The Volatility of Temperature, Pricing of Weather Derivatives, and Hedging Spatial Temperature Risk

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Overview of the presentation

1. The temperature market
2. A stochastic model for daily temperature
   - Continuous-time AR(p) model
   - with seasonal volatility
3. Temperature futures
   - HDD, CDD and CAT futures prices
   - The Samuelson effect
4. Hedging spatial temperature risk
   - Constructing synthetic futures
   - Spatio-temporal temperature models

The temperature market

The temperature market

- Chicago Mercantile Exchange (CME) organizes trade in temperature derivatives:
  - Futures contracts on monthly and seasonal temperatures
  - European call and put options on these futures
- Contracts on 18 US, 6 Canadian, 2 Japanese and 9 European cities
  - Stockholm

HDD and CDD

- HDD (heating-degree days) over a period $[\tau_1, \tau_2]$

$$\int_{\tau_1}^{\tau_2} \max(18 - T(u), 0)\, du$$

- HDD is the accumulated degrees when temperature $T(u)$ is below 18
- CDD (cooling-degree days) is correspondingly the accumulated degrees when temperature $T(u)$ is above 18

$$\int_{\tau_1}^{\tau_2} \max(T(u) - 18, 0)\, du$$

CAT and PRIM

- CAT = cumulative average temperature
  - Average temperature here meaning the *daily* average

$$\int_{\tau_1}^{\tau_2} T(u)\,du$$

- PRIM = Pacific Rim, the average temperature

$$\frac{1}{\tau_2 - \tau_1}\int_{\tau_1}^{\tau_2} T(u)\,du$$

At the CME...

- Futures written on HDD, CDD, CAT and PRIM as index
  - HDD and CDD is the index for US temperature futures
  - CAT index for European temperature futures, along with HDD and CDD
  - PRIM only for Japan
- Discrete (daily) measurement of HDD, CDD, CAT and PRIM
- All futures are cash settled
  - 1 trade unit=20 Currency (trade unit being HDD, CDD or CAT)
  - Currency equal to USD for US futures and GBP for European
- Call and put options written on the different futures

A stochastic model for temperature

A continuous-time AR(p)-process

- Define the Ornstein-Uhlenbeck process $\mathbf{X}(t) \in R^p$

$$d\mathbf{X}(t) = A\mathbf{X}(t)\,dt + \mathbf{e}_p(t)\sigma(t)\,dB(t)\,,$$

- $\mathbf{e}_k$: k'th unit vector in R^p
- $\sigma(t)$: temperature "volatility"
- A: $p \times p$-matrix

$$A = \begin{bmatrix} \mathbf{0} & & \mathbf{I} \\ -\alpha_p & \cdots & -\alpha_1 \end{bmatrix}$$

- Explicit solution of $\mathbf{X}(t)$:

$$\mathbf{X}(s) = \exp\left(A(s-t)\right)\mathbf{x} + \int_t^s \exp\left(A(s-u)\right)\mathbf{e}_p\sigma(u)\,dB(u)\,,$$

- Temperature dynamics $T(t)$ defined as

$$T(t) = \Lambda(t) + X_1(t)$$

- $X_1(t)$ CAR(p) model, $\Lambda(t)$ seasonality function
- Temperature will be normally distributed at each time

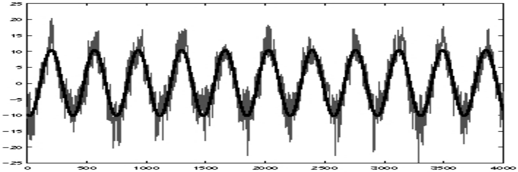
Why is X_1 a CAR(p) process?

- Consider $p = 3$
- Do an Euler approximation of the $\mathbf{X}(t)$-dynamics with time step 1
  - Substitute iteratively in $X_1(t)$-dynamics
  - Use $B(t+1) - B(t) = \epsilon(t)$
- Resulting discrete-time dynamics

$$X_1(t+3) \approx (3-\alpha_1)X_1(t+2) + (2\alpha_1 - \alpha_2 - 1)X_1(t+1) \\ + (\alpha_2 - 1 + (\alpha_1 + \alpha_3))X_1(t) + \sigma(t)\epsilon(t)\,.$$

Stockholm temperature data

- Daily average temperatures from 1 Jan 1961 till 25 May 2006
  - 29 February removed in every leap year
  - 16,570 recordings
- Last 11 years snapshot with seasonal function



- Fitting of model goes stepwise:
  1. Fit seasonal function $\Lambda(t)$ with least squares
  2. Fit AR(p)-model on deseasonalized temperatures
  3. Fit seasonal volatility $\sigma(t)$ to residuals
- We focus on the last two steps
  - Supposing a seasonal function

$$\Lambda(t) = a_0 + a_1\, t + a_2 \cos\left(2\pi(t - a_3)/365\right)$$

2. Fitting an auto-regressive model

- Remove the effect of $\Lambda(t)$ from the data

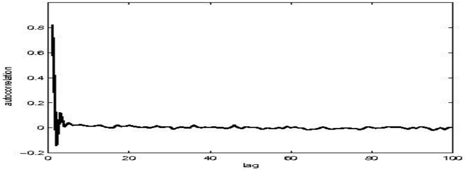
$$Y_i := T(i) - \Lambda(i)\,, i = 0, 1, \ldots$$

- Claim that AR(3) is a good model for Y_i:

$$Y_{i+3} = \beta_1 Y_{i+2} + \beta_2 Y_{i+1} + \beta_3 Y_i + \sigma_i \epsilon_i\,,$$

- The partial autocorrelation function for the data suggest AR(3)

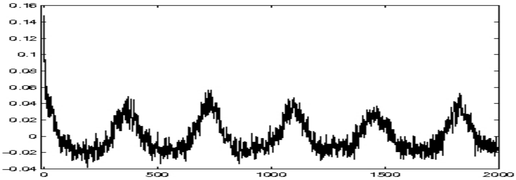
$$Y_i := T(i) - \Lambda(i)\,, i = 0, 1, \ldots$$



- Estimates $\beta_1 = 0.957, \beta_2 = -0.253, \beta_3 = 0.119$ (significant at 1% level)

3. Seasonal volatility

- Consider the residuals from the AR(3) model
- Close to zero ACF for residuals
- Highly seasonal ACF for *squared* residuals

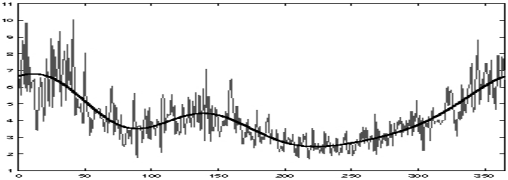


- Suppose the volatility is a truncated Fourier series

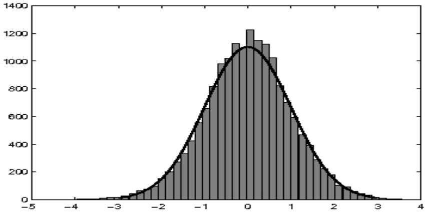
$$\sigma^2(t) = c + \sum_{i=1}^{4} c_i \sin(2i\pi t/365) + \sum_{j=1}^{4} d_j \cos(2j\pi t/365)$$

- This is calibrated to the daily variances
  - 45 years of daily residuals
  - Line up each year next to each other
  - Calculate the variance for each day in the year

- A plot of the daily empirical variance with the fitted squared volatility function
- High variance in winter, and early summer
- Low variance in spring and late summer/autumn



- Dividing out the seasonal volatility from the regression residuals
- ACF for squared residuals non-seasonal
  - ACF for residuals unchanged
  - Residuals become (close to) normally distributed



Temperature futures

Some generalities on temperature futures

- HDD-futures price $F_{\text{HDD}}(t, \tau_1, \tau_2)$ at time $t \leq \tau_1$
  - No trade in settlement period

$$0 = e^{-r(\tau_2 - t)} \mathbb{E}_Q \left[\int_{\tau_1}^{\tau_2} \max(c - T(u), 0)\, du - F_{\text{HDD}}(t, \tau_1, \tau_2) \,|\, \mathcal{F}_t \right].$$

- Constant interest rate r, and settlement at the end of index period, τ_2
- Q is a risk-neutral probability
  - Not unique since market is incomplete
  - Temperature (and HDD) is not tradeable
- c is equal to 65°F or 18°C

- Adaptedness of $F_{\mathrm{HDD}}(t,\tau_1,\tau_2)$ yields

$$F_{\mathrm{HDD}}(t,\tau_1,\tau_2)=\mathbb{E}_Q\left[\int_{\tau_1}^{\tau_2}\max(c-T(u),0)\,du\,|\,\mathcal{F}_t\right]$$

- Analogously, the CDD and CAT futures prices are

$$F_{\mathrm{CDD}}(t,\tau_1,\tau_2)=\mathbb{E}_Q\left[\int_{\tau_1}^{\tau_2}\max(T(u)-c,0)\,du\,|\,\mathcal{F}_t\right]$$

$$F_{\mathrm{CAT}}(t,\tau_1,\tau_2)=\mathbb{E}_Q\left[\int_{\tau_1}^{\tau_2}T(u)\,du\,|\,\mathcal{F}_t\right]$$

A class of risk neutral probabilities

- Parametric sub-class of risk-neutral probabilities Q^{θ}
- Defined by Girsanov transformation of $B(t)$

$$dB^{\theta}(t) = dB(t) - \theta(t)\,dt$$

- $\theta(t)$ deterministic *market price of risk*
- Dynamics of $\mathbf{X}(t)$ under Q^{θ}:

$$d\mathbf{X}(t) = \left(A\mathbf{X}(t) + \mathbf{e}_p\sigma(t)\theta(t)\right)\,dt + \mathbf{e}_p\sigma(t)\,dB^{\theta}(t)\,.$$

- Feasible dynamics for explicit calculations

CDD futures

- CDD-futures price

$$F_{\mathrm{CDD}}(t,\tau_1,\tau_2)=\int_{\tau_1}^{\tau_2} v(t,s)\Psi\left(\frac{m(t,s,\mathbf{e}_1'\exp(A(s-t))\mathbf{X}(t))}{v(t,s)}\right)ds$$

where

$$m(t,s,x)=\Lambda(s)-c+\int_t^s \sigma(u)\theta(u)\mathbf{e}_1'\exp(A(s-u))\mathbf{e}_p\,du+x$$

$$v^2(t,s)=\int_t^s \sigma^2(u)\left(\mathbf{e}_1'\exp(A(s-u))\mathbf{e}_p\right)^2\,du$$

- $\Psi(x)=x\Phi(x)+\Phi'(x)$, Φ being the cumulative standard normal distribution function.

- The futures price is dependent on $\mathbf{X}(t)$
  - In discrete-time, the futures price is a function of the lagged temperatures $T(t), T(t-1), \ldots, T(t-p)$
- Time-dynamics of the CDD-futures price

$$dF_{\mathrm{CDD}}(t,\tau_1,\tau_2) = \sigma(t)\int_{\tau_1}^{\tau_2} \mathbf{e}_1' \exp(A(s-t))\mathbf{e}_p \\ \times \Phi\left(\frac{m(t,s,\mathbf{e}_1'\exp(A(s-t))\mathbf{X}(t))}{v(t,s)}\right) ds\, dB^{\theta}(t)$$

CAT futures

- CAT-futures price

$$
\begin{aligned}
F_{\mathrm{CAT}}(t,\tau_1,\tau_2) &= \int_{\tau_1}^{\tau_2} \Lambda(u)\,du + \mathbf{a}(t,\tau_1,\tau_2)\mathbf{X}(t) \\
&+ \int_t^{\tau_1} \theta(u)\sigma(u)\mathbf{a}(t,\tau_1,\tau_2)\mathbf{e}_p\,du \\
&+ \int_{\tau_1}^{\tau_2} \theta(u)\sigma(u)\mathbf{e}_1' A^{-1}\left(\exp\left(A(\tau_2-u)\right) - I_p\right)\mathbf{e}_p\,du
\end{aligned}
$$

with I_p being the $p \times p$ identity matrix and

$$
\mathbf{a}(t,\tau_1,\tau_2) = \mathbf{e}_1' A^{-1}\left(\exp\left(A(\tau_2-t)\right) - \exp\left(A(\tau_1-t)\right)\right)
$$

- Time-dynamics of F_{CAT}

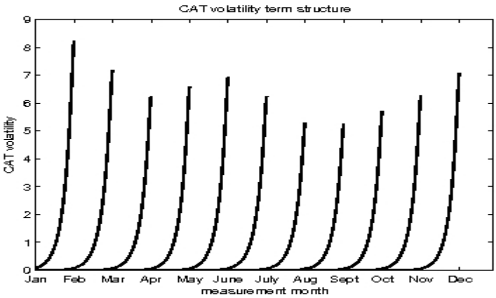
$$dF_{\mathrm{CAT}}(t,\tau_1,\tau_2) = \Sigma_{\mathrm{CAT}}(t,\tau_1,\tau_2)\, dB^{\theta}(t)$$

where

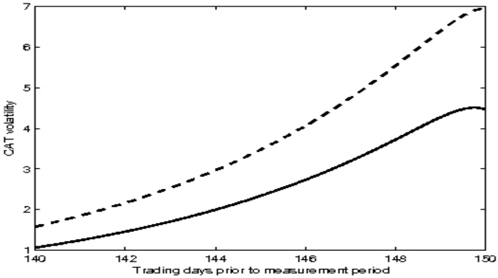
$$\Sigma_{\mathrm{CAT}}(t,\tau_1,\tau_2) = \sigma(t)\mathbf{e}_1' A^{-1}\left(\exp\left(A(\tau_2 - t)\right) - \exp\left(A(\tau_1 - t)\right)\right)\mathbf{e}_p$$

- Σ_{CAT} is the CAT volatility term structure

- Seasonal volatility, with maturity effect
- Plot of the volatility term structure as a function of t up till delivery
    - Monthly contracts
    - Parameters taken from Stockholm for CAR(3)



- The Samuelson effect
  - The volatility is decreasing with time to delivery
  - Typical in mean-reverting markets
- AR(3) has memory
  - Implies a modification of this effect
  - Plot shows volatility of CAT with monthly vs. weekly measurement period



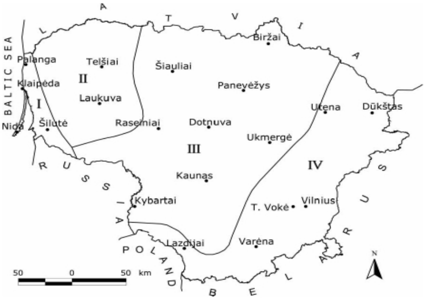
Hedging spatial temperature risk

The spatial hedging problem

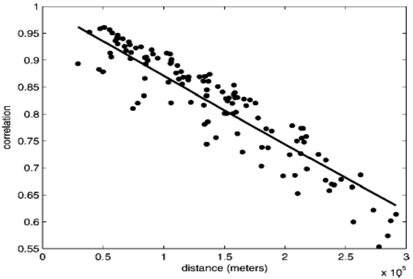
- Temperature futures used to remove temperature risk
  - Exchange varying temperature (index) by a fixed temperature (index)
- Temperature futures available only in specific locations (cities)
- An investor may want a temperature futures at a certain location not offered in the market
  - ..or a futures on the average temperature over an area
- Q: How to design an optimal futures based on the traded ones in the market?
  - Requires a spatial model for temperature

A spatial-temporal temperature model

- Motivation from a study of Lithuanian temperatures
- Data series for more than 40 years available in 16 stations



- Analysis of CAR(1) (OU)-process for each location
- Empirical findings:
    1. Seasonality function similar for the different locations
    2. Speed of mean-reversion α reasonably stable over locations
    3. Seasonal volatility similar over locations
    4. Clear spatial correlation structure in residuals



Spatial-temporal dynamics

$$dT(t,x) = d\Lambda(t,x) - \alpha(x)\,(T(t,x) - \Lambda(t,x))\,dt + \sigma(t,x)\,dW(t,x)$$

- $W(t,\cdot)$ is an $L^2(\mathcal{D})$-valued Wiener process
- Continuous spatial covariance function $q(x,y)$
  - Strictly positive definite
  - symmetric
- Define operator Q on $L^2(\mathcal{D})$ with integral kernel q
- Expansion for W in terms of the eigenvalues and vectors of Q

$$W(t,\cdot) = \sum_{i=1}^{\infty} \sqrt{\lambda_i} B_i(t) e_i$$

Optimal synthetic futures

- Given a temperature index $\mathcal{I}(x_i)$ in different locations $x_1, \ldots, x_n$
  - Locations where futures on $\mathcal{I}(x_i)$ are traded
  - $\mathcal{I}$ may be CDD, HDD, CAT
  - Mixtures of these, and even different measurement periods
- Problem: Find optimal (adapted) strategy $\mathbf{a}(t)$ minimizing

$$\mathbb{E}\left[\left(\mathcal{I}(y) - \sum_{i=1}^{n} a_i(t)\mathcal{I}(x_i)\right)^2 \mid \mathcal{F}_t\right]$$

- y is the location where we would like to have the temperature futures on the index $\mathcal{I}$

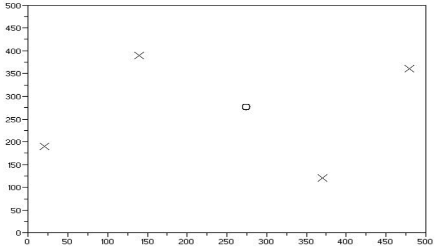
Example

- Suppose temperature model with no spatial dependency in Λ, α and σ
- Spatial dependency modelled by a spherical correlation function

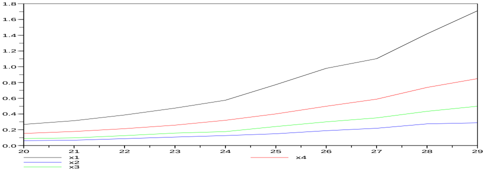
$$q(x,y) = 1 - \frac{3}{2}\frac{|x-y|}{\gamma} + \frac{1}{2}\frac{|x-y|^3}{\gamma^3}$$

- All parameters taken from the Lithuanian study
  - Average values

- Set-up with 4 locations around a point y
- CAT indices, with 10 measurement days in middle of June
- Calculate $a_1(t), \ldots, a_4(t)$ for 10 previous days to measurement
- Based on simulation of the temperature field



- Average weights are: $a_1 = 0.08$, $a_2 = 0.37$, $a_3 = 0.35$ and $a_4 = 0.21$
- Plot of standard deviation of weights relative to mean
    - Plotted in %



- Note the increase, similar to the volatility of CATs
- Also, the variation dependent on distance to y
    - Note: more tradeable futures do not necessarily reduce risk
    - Reduction depends on correlation and geometry of the locations

Conclusions

- CAR(p) model for the temperature dynamics
  - Auto-regressive process, with
  - Seasonal mean
  - seasonal volatility
- Allows for analytical futures prices for the traded contracts on CME
  - HDD/CDD, CAT and PRIM futures
  - Futures contracts with delivery over months or seasons
  - Seasonal volatility with a modified Samuelson effect: volatility may even decrease close to maturity
- Considered the construction of a synthetic temperature futures based on traded contracts
  - Minimizing the variance
  - Based on a spatio-temporal temperature model

Coordinates

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- folk.uio.no/fredb
- www.cma.uio.no

References

Alaton, Djehiche, and Stillberger. On modelling and pricing weather derivatives. *Appl. Math. Finance*, 9, 2002

Benth and Saltyte-Benth. Stochastic modelling of temperature variations with a view towards weather derivatives. *Appl. Math. Finance*, 12, 2005

Barth, Benth and Potthoff. Hedging of spatial temperature risk with markets-traded futures. E-print University of Oslo, 2007.

Benth and Saltyte-Benth. Stochastic modelling of temperature variations with a view towards weather derivatives. *Appl. Math. Finance*, 12, 2005

Benth and Saltyte-Benth. The volatility of temperature and pricing of weather derivatives. *Quantit. Finance*, 7, 2007

Saltyte Benth, Benth and Jalinskas. A spatial-temporal model for temperature with seasonal variance. *J. Appl. Statist.*, 34, 2007

Brody, Syroka and Zervos. Dynamical pricing of weather derivatives. *Quantit. Finance*, 3, 2002

Campbell and Diebold. Weather forecasting for weather derivatives. *J. Amer. Stat. Assoc.*, 100, 2005

Dornier and Querel. Caution to the wind. *Energy Power Risk Manag.*, August, 2000