

Pricing of electricity forwards

– The risk premium –

Fred Espen Benth

In collaboration with Alvaro Cartea (London), Rüdiger Kiesel (Ulm) and Thilo Meyer-Brandis (Oslo/Munich)

Centre of Mathematics for Applications (CMA)
University of Oslo, Norway

Seminar, Carnegie Mellon University, February 23 2009

Introduction

- Problem: what is the connection between spot and forward prices in electricity?
- Electricity is a non-storable commodity
- How to explain the risk premium?
 - Empirical and economical evidence: Sign varies with time to delivery
- Propose two approaches:
 1. Information approach
 2. Equilibrium approach
- Purpose: try to explain the risk premium for electricity

Outline of talk

1. Example of an electricity market: NordPool
2. The “classical” spot-forward relation
3. The information approach
4. The equilibrium approach
5. Conclusions

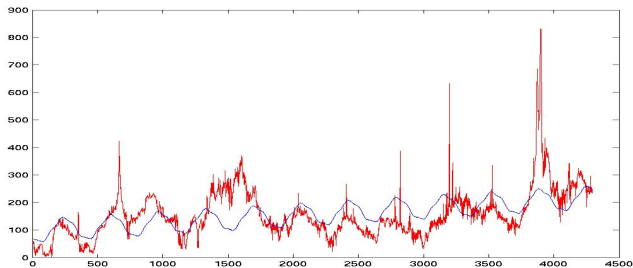
Example of an electricity market: NordPool

- The NordPool market organizes trade in
 - Hourly spot electricity, next-day delivery
 - Financial forward contracts
 - In reality mostly futures, but we make no distinction here
 - European options on forwards
- Difference from “classical” forwards:
 - Delivery over a period rather than at a fixed point in time

Elspot: the spot market

- A (non-mandatory) hourly market with physical delivery of electricity
- Participants hand in bids before noon *the day ahead*
 - Volume and price for each of the 24 hours next day
 - Maximum of 64 bids within technical volume and price limits
- NordPool creates demand and production curves for the next day before 1.30 pm

- The *system price* is the equilibrium
 - Reference price for the forward market
- Historical system price from the beginning in 1992
 - note the spikes....



The forward market

- Forward with delivery over a period
- Financial market
- Settlement with respect to system price in the delivery period
- Delivery periods
 - Next day, week or month
 - Quarterly (earlier seasons)
 - Yearly
- Overlapping settlement periods (!)
- Contracts also called *swaps*: Fixed for floating price

The option market

- European call and put options on electricity forwards
 - Quarterly and yearly electricity forwards
- Low activity on the exchange
- OTC market for electricity derivatives huge
 - Average-type (Asian) options, swing options

The spot-forward relation

The spot-forward relation: some “classical” theory

- The **no-arbitrage** forward price (based on the buy-and-hold strategy)

$$F(t, T) = S(t)e^{r(T-t)}$$

- A risk-neutral expression of the price as

$$F(t, T) = \mathbb{E}_Q [S(T) | \mathcal{F}_t]$$

- The **risk premium** is defined as

$$R(t, T) = F(t, T) - \mathbb{E}[S(T) | \mathcal{F}_t]$$

- In the case of electricity:
 - Storage of spot is *not* possible (only indirectly in water reservoirs)
 - Buy-and-hold strategy fails
 - No foundation for the “classical” spot-forward relation
 - ...and hence no rule for what Q should be!
- Thus: What is the link between $F(t, T)$ and $S(t)$?

Economical “intuition” for electricity

- Short-term *positive* risk premium
 - Retailers (consumers) hedge “spike risk”
 - Spikes lead to expensive electricity
 - Accept to pay a premium for locking in prices in the short-term
- Long-term *negative* risk premium
 - Producers hedge their future production
 - Long-term contracts (quarters/years)
- The market may have a change in the sign of the risk premium

Empirical evidence for electricity

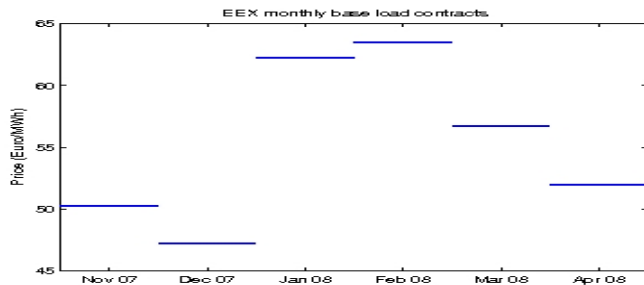
- Longstaff & Wang (2004), Geman & Vasicek : PJM market
 - Positive premium in the short-term market
- Diko, Lawford & Limpens (2006)
 - Study of EEX, PWN, APX, based on multi-factor models
 - Changing sign of the risk premium
- Kolos & Ronn (2008)
 - Market price of risk: expected risk-adjusted return
 - Multi-factor models
 - Negative on the short-term, positive on the long term

- Explore two possible approaches to price electricity futures
 1. The information approach based on market forecasts
 2. An equilibrium approach based on market power of the consumers and producers
- For simplicity we first restrict our attention to $F(t, T)$
 - Electricity forwards deliver over a time period
 - Creates technical difficulties for most spot models
 - Ignore this here
 - In the equilibrium approach we consider delivery periods

The information approach

The information approach: idea

- Idea is the following:
 - Electricity is non-storable
 - Future predictions about market will not affect current spot
 - However, it will affect forward prices
- Stylized example:
 - Planned outage of a power plant in one month
 - Will affect forwards delivering in one month
 - But *not* spot today
- Market example
 - In 2007 market knew that in 2008 CO₂ emission costs will be introduced
 - No effect on spot prices in the EEX market in 2007
 - However, clear effect on the forward prices around New Year



The information approach: definition

- Define the forward price as

$$F_{\mathcal{G}}(t, T) = \mathbb{E}[S(T) | \mathcal{G}_t]$$

- \mathcal{G}_t includes spot information up to current time (\mathcal{F}_t) and forward-looking information
- The **information premium**

$$I_{\mathcal{G}}(t, T) = F_{\mathcal{G}}(t, T) - \mathbb{E}[S(T) | \mathcal{F}_t]$$

- Rewrite the information premium using double conditioning and $\mathcal{F}_t \subset \mathcal{G}_t$

$$I_{\mathcal{G}}(t, T) = \mathbb{E}[S(T) | \mathcal{G}_t] - \mathbb{E}[\mathbb{E}[S(T) | \mathcal{G}_t] | \mathcal{F}_t]$$

- The information premium is the residual random variable after projecting $F_{\mathcal{G}}(t, T)$ onto $L^2(\mathcal{F}_t, P)$
 - $I_{\mathcal{G}}$ measures how much more information is contained in \mathcal{G}_t compared to \mathcal{F}_t

- Note that

$$\mathbb{E}[l_{\mathcal{G}}(t, T) | \mathcal{F}_t] = 0$$

- $l_{\mathcal{G}}(t, T)$ is orthogonal to $R(t, T)$
 - The risk premium $R(t, T)$ is \mathcal{F}_t -adapted
- Thus, impossible to obtain a given $l_{\mathcal{G}}(t, T)$ from an appropriate choice of Q in $R(t, T)$
 - Including future information creates new ways of explaining risk premia

Example: temperature predictions

- Temperature dynamics

$$dY(t) = \gamma(\mu(t) - Y(t)) dt + \eta dB(t)$$

- Spot price dynamics

$$dS(t) = \alpha(\lambda(t) - S(t)) dt + \sigma\rho dB(t) + \sigma\sqrt{1 - \rho^2} dW(t)$$

- ρ is the correlation between temperature and spot price
 - NordPool: $\rho < 0$, since high temperature implies low prices, and vice versa

- Suppose we have some temperature forecast at time T_1
 - Full, or at least some, knowledge of $Y(T_1)$

$$\mathcal{F}_t \subset \mathcal{G}_t \subset \mathcal{H}_t \triangleq \mathcal{F}_t \vee \sigma(Y(T_1))$$

- We want to compute (for $T \leq T_1$)

$$F_{\mathcal{G}}(t, T) = \mathbb{E}[S(T) | \mathcal{G}_t]$$

- Program:
 1. Find a Brownian motion wrt \mathcal{G}_t
 2. Compute the conditional expectation

- From the theory of “enlargement of filtrations”:
 - There exists a \mathcal{G}_t -adapted drift θ_1 such that \tilde{B} is a \mathcal{G}_t -Brownian motion,

$$d\tilde{B}(t) = dB(t) - \theta_1(t) dt$$

- The drift is expressed as

$$\theta_1(t) = a_1(t) \left(e^{\gamma T_1} \mathbb{E}[Y(T_1) | \mathcal{G}_t] - e^{\gamma t} Y(t) - \gamma \int_t^{T_1} \mu(u) e^{\gamma u} du \right)$$

$$a_1(t) = \frac{2\gamma e^{\gamma t}}{\eta(e^{2\gamma T_1} - e^{2\gamma t})}$$

- Dynamics of S in terms of \tilde{B} :

$$dS(t) = \alpha \left(\rho \frac{\sigma}{\alpha} \theta_1(t) + \lambda(t) - S(t) \right) dt + \sigma \rho d\tilde{B}(t) + \sigma \sqrt{1 - \rho^2} dW(t)$$

- Note that we have a mean-reversion level being *stochastic*
 - Explicitly dependent on the temperature prediction and today's temperature
- $\theta_1(t)$ is the **market price of information**, or **information yield**

- Calculate the forward price

$$\begin{aligned} F_G(t, u) &= \mathbb{E}[S(u) | \mathcal{F}_t] + I_G(t, T) \\ &= S(t) \exp(-\alpha(T-t)) + \alpha \int_t^T \lambda(s) e^{-\alpha(T-s)} ds + I_G(t, T) \end{aligned}$$

- The information premium is, by applying the definition

$$I_G(t, T) = \rho\sigma \mathbb{E} \left[\int_t^T e^{-\alpha(T-s)} dB(s) | \mathcal{G}_t \right]$$

- Use that \tilde{B} is a \mathcal{G}_t -Brownian motion

- Expression for the information premium

$$I_{\mathcal{G}}(t, T) =$$

$$\rho A(t, T) \left(e^{\gamma T_1} \mathbb{E}[Y(T_1) | \mathcal{G}_t] - e^{\gamma t} Y(t) - \gamma \int_t^{T_1} \mu(s) e^{\gamma s} ds \right)$$

where

$$A(t, T) = \frac{2\gamma\sigma e^{\gamma T} (1 - e^{-(\alpha+\gamma)(T-t)})}{\eta(\alpha + \gamma)(e^{2\gamma T_1} - e^{2\gamma t})}$$

- Observe that $A(t, T)$ is positive
- The sign of the information premium is determined by
 - The correlation ρ
 - The temperature prediction

Example with complete information

- Suppose we know the temperature at T_1
 - The information set is \mathcal{H}_t
 - Unlikely situation of perfect future knowledge....
- Assume we we expect a temperature drop

$$Y(T_1) < e^{-\gamma(T_1-t)} Y(t) + \gamma \int_t^{T_1} \mu(s) e^{-\gamma(T_1-s)} ds$$

- At NordPool, where $\rho < 0$:
 - The information premium is positive
- Drop in temperature will lead to increasing demand, and thus higher prices

The equilibrium approach

The equilibrium approach: idea

- Producers and consumers can trade in both spot and forward markets
 - No speculators in our set-up
- We suppose that the forwards deliver electricity over an agreed period
 - No fixed delivery time as in other commodity markets
 - Natural for electricity due to its nature
- Choice of an electricity producer
 - Sell production on spot market, *or* on the forward market

- Producer is indifferent when (U_{pr} is the utility function)

$$\mathbb{E} \left[U_{pr} \left(\int_{\tau_1}^{\tau_2} S(u) du \right) \right] = \mathbb{E} [U_{pr} ((\tau_2 - \tau_1) F_{pr}(t, \tau_1, \tau_2))]$$

- The certainty equivalence principle
- F_{pr} is the **lowest** acceptable price for the producer can accept to be interested in entering a forward
 - Similarly, F_c is the highest acceptable price for the consumer, for a given utility function U_c
- We assume exponential utility $U(x) = 1 - \exp(-\gamma x)$, with respective risk aversion for producer and consumer γ_{pr} and γ_c

- By Jensen's inequality, the predicted average spot price is within the price bounds

$$F_{\text{pr}}(t, \tau_1, \tau_2) \leq \mathbb{E} \left[\frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} S(u) du \mid \mathcal{F}_t \right] \leq F_c(t, \tau_1, \tau_2)$$

- Hypothesis: The settlement price of the forward will depend on the market power $p \in [0, 1]$ of the producer

$$F^P(t, \tau_1, \tau_2) = pF_c(t, \tau_1, \tau_2) + (1 - p)F_{\text{pr}}(t, \tau_1, \tau_2)$$

- Assume a simple two-factor spot model with jump component

$$S(t) = \Lambda(t) + X(t) + Y(t)$$

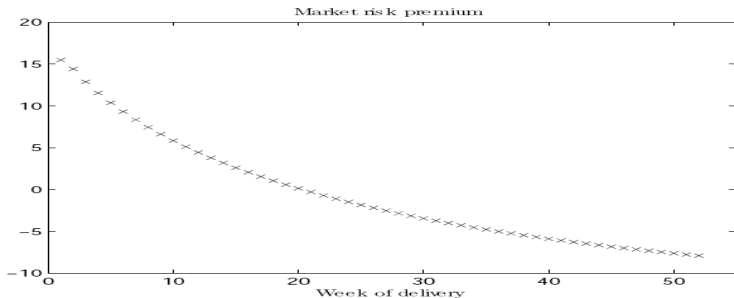
- $\Lambda(t)$ seasonal function

$$dY(t) = -\lambda Y(t) dt + Z dN(t)$$

- Jumps (accounting for spikes)
 - Z jump size
 - N Poisson process
- Slowly varying base component

$$dX(t) = -\alpha X(t) dt + \sigma dB(t)$$

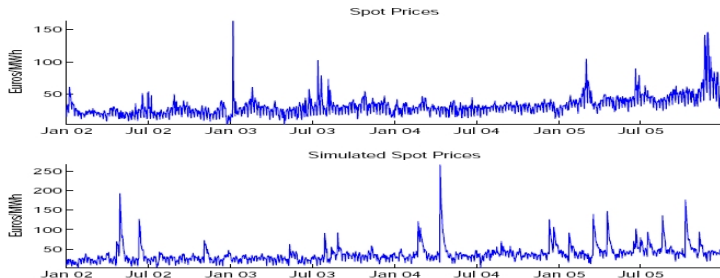
- Calculate prices for weekly contracts and compute the risk premium
 - The market power set to $p = 0.25$
 - Constant positive jumps at rate 2/year



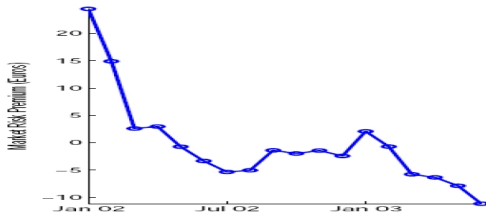
- Note the **positive** risk premium in the short end
 - Caused by the jump risk

Empirical example: EEX (Metka, Ulm)

- Fit two-factor model to daily EEX spot prices (Jan 02 – Dec 05)



- Using observed prices for 18 monthly forward contracts and fitted spot model
 - Calculate the risk premium,
 - Difference between forward price and predicted spot
 - Observe a positive premium in the short end, and negative in the long end



- Based on all available forward prices in the study, risk aversion parameters were determined

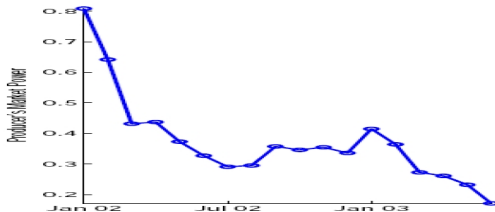
- $\gamma_{pr} \geq 0.421$ and $\gamma_c \geq 0.701$ are such that

$$F_{pr}(t, \tau_1, \tau_2) \leq F(t, \tau_1, \tau_2) \leq F_c(t, \tau_1, \tau_2)$$

- Calculate the empirical market power

$$p(t, \tau_1, \tau_2) = \frac{F(t, \tau_1, \tau_2) - F_{pr}(t, \tau_1, \tau_2)}{F_c(t, \tau_1, \tau_2) - F_{pr}(t, \tau_1, \tau_2)}$$

- Observe that producer's power is strong in the short end, while decreasing to be rather weak in the long end



Conclusions

- Discussed two potential ways to understand the link between spot and forward prices in electricity markets
- Information approach:
 - Include future information in pricing
- Equilibrium approach:
 - Certainty equivalence principle for upper and lower bounds of prices
 - Use market power as an explanatory variable for price formation

Coordinates

- fredb@math.uio.no
- folk.uio.no/fredb
- www.cma.uio.no

References

Benth and Meyer-Brandis (2007). The information premium in electricity markets. E-print

Benth, Cartea and Kiesel (2008). Pricing forward contracts in power markets by the certainty equivalence principle: explaining the sign of the market risk premium. *J Banking Finance*, 32