Utility Maximization with Unbounded Semimartingales: on the Supermartingale Property[(1)] and on the Indifference Price[(2)]

Marco Frittelli
Università degli studi di Firenze, Italy

We consider a stochastic financial incomplete market where the price processes are described by a vector valued semimartingale that is *not necessarily locally bounded.* We assume that the utility function is finite valued and smooth on the entire real line and satisfies Reasonable Asymptotic Elasticity. When the investor is willing to take more risk to really increase his/her expected utility in this very risky market, it is convenient to enlarge the set of allowed strategies by admitting losses bounded from below by $-cW$, where W is a positive random variable, possibly unbounded from above. In this general setting and under a compatibility condition on W, it was shown in Biagini-Frittelli (2005) that the optimal claim admits an integral representation as soon as the minimax $\sigma-$ martingale measure is equivalent to the reference probability measure.

(1) We show that the optimal wealth process is in fact a supermartingale with respect to every σ-martingale measure with finite generalized entropy, thus extending the analogous result proved by Schachermayer (2001) in the locally bounded case.

(2) In a recent work, we extended the above mentioned analysis by adopting a weaker compatibility condition (on W) that allows considering a larger class of market models. Here we further extend this approach and face the problem of exponential utility maximization with a random endowment. We then apply the duality relation to compute the indifference price of a claim satisfying weak integrability conditions.

The indifference price leads to a convex risk measure whose dual representation is based on a set of singular functionals which belong to the dual space of an appropriate Orlicz space. The penalty term is split into an entropic component and a singular one that is interpreted as a measure of catastrophic events.

(1) Based on a joint work with S. Biagini
(2) Based on a joint work with S. Biagini, M. Grasselli, T. Hurd.