Research Statement

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My research interest lies in applied mathematics and nonlinear partial differential equations, particularly in problems with applications to fluid dynamics. I am drawn to problems with interesting physics, challenging mathematics and those with significant applications. My Ph.D. work has given me an opportunity to study a problem theoretically, numerically and even experimentally. In addition, I have also worked on some other problems in applied mathematics. I have described some of these problems below.

1 Mathematical Analysis of Bodies Sedimenting in Newtonian and Non-Newtonian Fluids.

Over the last few years, I have worked in the field of mathematical fluid dynamics. My thesis work has an interdisciplinary flavor and could be placed under the subject of fluid-structure interaction problems. The objective of my thesis is to study the rather interesting phenomenon regarding the orientation of symmetric rigid bodies falling in Newtonian and Viscoelastic fluids. This work has applications in fields where sedimentation or transport phenomenon is of interest such as in biomechanics, where the process of separation of biomolecules via electrophoresis involves sedimentation of particles through organic media [8] and in Material Science, where orientation of short fiber like particles in a polymer network is important for enhancing the mechanical properties of composite materials (see [15]). One could also point to applications in hemodynamics, where transport and orientation of blood cells in the plasma can be of significant interest or in the environmental sciences where removal of particulates to reduce air and water pollution is of major concern. However, the immediate motivation for this problem comes from the experimental work of D.D. Joseph [11], [12], [13], [16], [9] on flow-induced microstructures. The key to understanding the phenomenon of *drafting, kissing*, and *tumbling* and the symmetries adopted by the interaction of spheres in non-Newtonian fluids lies in understanding the orientations of long bodies.

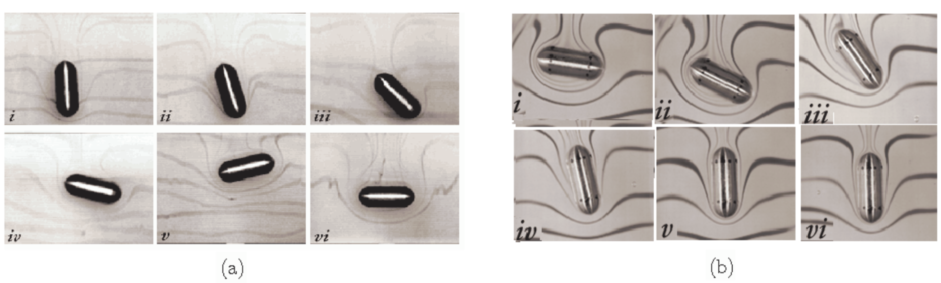

Figure 1: Terminal orientation of cylinders in (a) Newtonian and (b) viscoelastic liquids.(Courtsey of Prof. D.D. Joseph)

It is a well established fact that homogeneous bodies of revolution around an axis (call it a) with fore-aft symmetry will orient themselves with respect to the direction of gravity ($\mathbf{g}$) depending upon their shape and upon the nature of the fluid in which they are immersed. If, for instance, we are considering an ellipsoidal object falling in a Newtonian fluid such as water, then the body falls with a eventually becoming perpendicular to the direction of $\mathbf{g}$. However if the same body falls in a viscoelastic fluid where the inertial effects can be disregarded then a will eventually become parallel to $\mathbf{g}$. Furthermore, it has also been observed that elongated bodies falling in fluids with certain polymeric concentrations can taken on angles between the horizontal and vertical orientations. These intermediate angles are referred to in the literature as *tilt angles* [3], [2]. In my thesis, I have undertaken to explain these above mentioned phenomenon mathematically, computationally and experimentally.

- **Existence of steady states for motion of bodies in non-Newtonian fluids**[19, 21]: Existence for steady freefall in an unbounded Newtonian liquid has already been shown, however this problem remains unsolved in non-Newtonian liquid. Therefore we choose to fill this gap in the literature by choosing to work with the simplest model for a viscoelastic liquid, namely a second-order fluid model. This amounts to studying the following equations.

$$\begin{aligned} -\Delta u + \nabla p - \mathrm{We}\; u \cdot \nabla \Delta u &= f + \mathrm{We} \nabla \cdot [(\nabla u^T) A_1 + (1+\epsilon) {A_1}^2] \\ \mathrm{div}\; u &= 0, \quad u = 0 \text{ on } \Sigma \end{aligned} \tag{1}$$

$$\begin{aligned} \lim_{x \to \infty} (u + u_\infty) &= 0 \\ u_\infty &= \xi + \omega \times x \end{aligned}$$

  where A_1 is the symmetric part of the velocity gradient, u_∞ represents the rigid body motion with ξ the constant translational motion and ω, the rotational motion, We refers to the Weissenberg number and ϵ is a material parameter depending upon the normal stress coefficients. Since the problem involves also the motion of the body, we must, in addition, specify equations for the body. These are given as forces and torques acting on the body imposed by the fluid

$$\begin{aligned} \int_\Sigma T(\omega, \pi) \cdot n &= mg \\ \int_\Sigma y \times T(\omega, \pi) \cdot n &= R \times g \\ \omega \times g &= 0. \end{aligned} \tag{2}$$

  Here, T is the Cauchy Stress tensor, g represents the gravity vector, m the mass and R is the vector from the geometric center of the body to the center of mass of the body. We have shown existence of solutions to this nonlinear, coupled system for small We and zero Re. The proof follows from the application of suitable fixed point arguments in rather complex function spaces. This part of the work establishes the well posedness of the model that we will analyze further to understand the steady state orientation of sedimenting bodies.

- **Theoretical analysis of terminal orientation of bodies in Newtonian and non-Newtonian fluids**[5, 6]: The second problem involves the theoretical formulation and resolution of the orientation of bodies in different fluids. For this case we consider the steady free fall in Newtonian, viscoelastic and shear-thinning fluids modeled by the Navier-Stokes equations, second-order fluid equations and the power-law fluid equations respectively. The principle objective of this portion of the research is to explain the orientation phenomenon as a result of competing torques caused by different aspects of the fluid upon the sedimenting body [10]. In other words, the terminal orientation can be seen to be the result of torques due to inertia, elasticity and shear-thinning and the body tends to orient in the direction of the dominant torque. We are able to formulate the problem at first order in Re and We, i.e. for small Reynolds and Weisseneberg numbers respectively. In addition, we have also investigated the stability of the terminal states. In order to establish stability of the steady orientation, we need to evaluate the torque imposed by the surrounding fluid upon the sdimenting body. This was accomplished by means of a fortran code that we wrote for this purpose. Our theoretical analysis, it turns out, is in perfect match with experimental observations.

- **Experiments on terminal orientation of particles in fluids**[20]: We have also performed extensive experiments, (a) on particle orientations in a sedimentation tank and also (b) with the particle held fixed in the center of a flow chamber setup. For this work, we also involved several undergraduates in our department, as part of their senior research project. Our work has involved designing and building the experimental setup and also conducting the experiments with a variety of liquids, Newtonian and viscoelastic, of different concentrations. Rheological properties of the liquids used were ascertained using a cone and plate rheometer. We successfully verified earlier results and also managed to discover some previously unknown phenomenon on the sedimentation of flat ended cylinders, which is consistent with our theoretical predictions.

2 Classical and Quantized Solutions of the Perturbed Wave Equation with Singular Kernel.

This work was done as part of my masters thesis research. In this work, we investigated the perturbed massless, wave equation

$$\Box\phi + \frac{n(n+2)}{(1+x^2)^2}\phi = 0 \tag{3}$$

containing an external potential term, in the Lorentzian metric which arises from a first order perturbation of the non-linear wave equation, where n refers to the space-time dimension. This problem provides an interesting example of a conformally invariant equation in quantum field theory where the potential term can be singular. The eventual objective was to study the quantization of the field, ϕ, which was not considered in the previous literature. We obtained the classical solution to the problem for arbitrary dimension, n and also, the solutions to the initial value and the inhomogeneous problems, specifically for the case $n = 2$. These results were motivated by our attempt at quantization which was approached both from Lagrangian and Hamiltonian perspectives. The former is based upon the method pioneered by Richard Feynman and the latter is based on the heuristic approach of Irving Segal for the quantization of hyperbolic systems. While, we could not find the quantized field that we were looking for, we managed to obtain some preliminary results regarding the quantization of the field, ϕ. This work resulted in in two publihsed papers [17] and [18].

I have already had the pleasure of collaborating with several mathematicians and engineers for my doctoral work. For the future I look forward to more fruitful collborations and to working with graduate and undergraduate students. I am eager to begin working on challenging and interesting mathematical problems several of which I have already identified.

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