Summer Research in Applied Mathematics

UNIVERSITY OF OXFORD

Winston Yin · July – August, 2015

When I arrived at Oxford by coach on 10th July, the sky was clear and the sun was shining. The air was cool, in stark contrast to the steaming weather in Nice, Southern France, where I had travelled the week prior. This was a good start to my 7-week research programme at the University of Oxford.

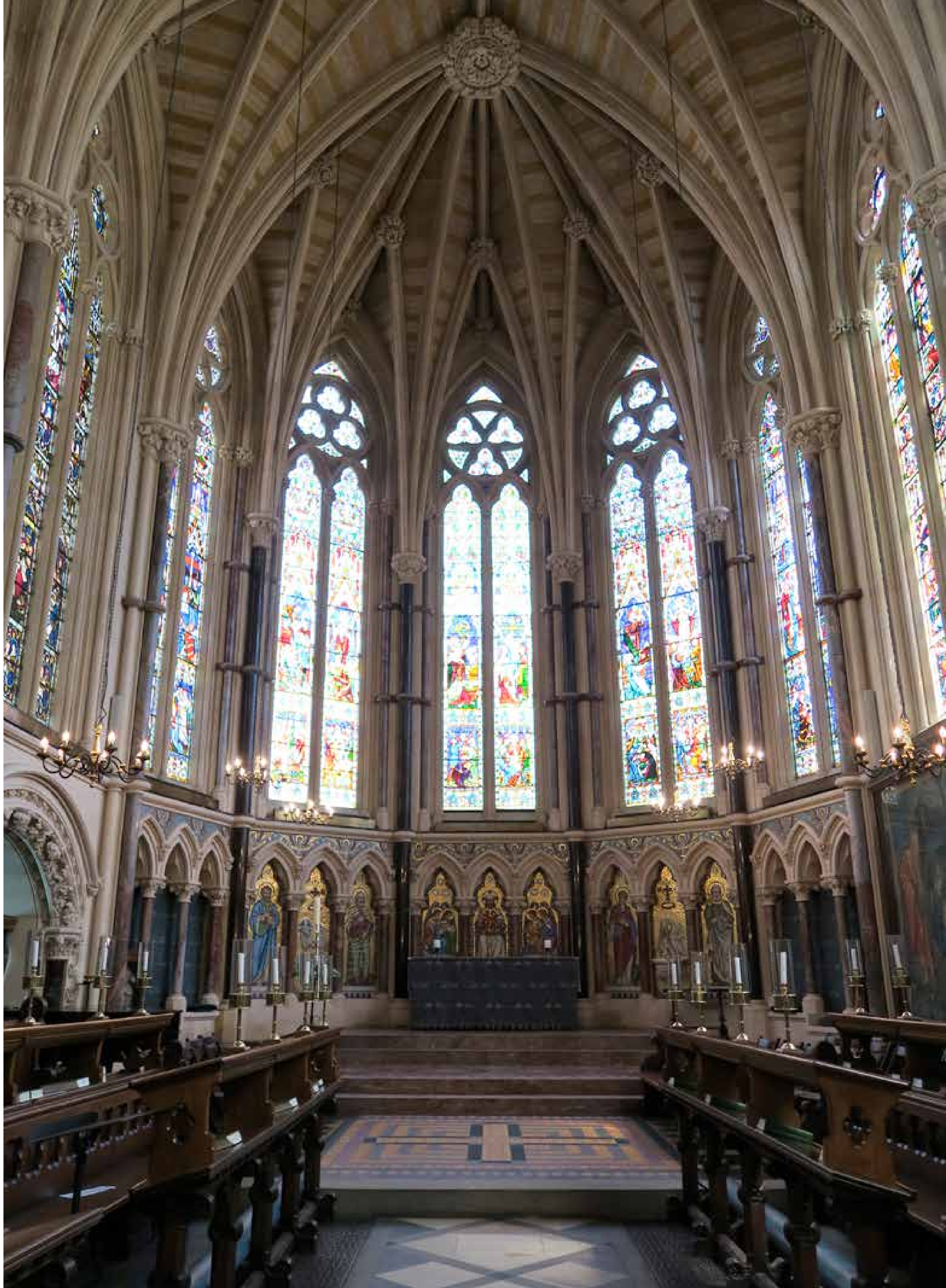
I was arranged to stay in Lincoln College housing, which, though surrounded by half-millenium-old buildings, was newly built and well furnished. I shared a suite with Thomas, another CMU student on a different project. The suite had a kitchen and a balcony, from which I would watch the stars and the Perseid meteor shower in August.

After settling down, I soon met my advisor, Anton Mühlemann, who works at the OxPDE group at the Mathematical Institute, located in the new Andrew Wiles building. I was given an office shared with a post-doc and a Malaysian undergraduate student, who was also in a summer programme. I immediately began working.

The building is modern and contained many tributes to former members of the institute, including Sir Roger Penrose's famed tiling, as seen in the beautiful floor pattern just outside the entrance.

Oxford is a tiny city, but as the Chinese saying goes: "small as it is, the sparrow has all the vital organs". It is home to around forty colleges, the first of which I visited was Exeter College. Matias, an old friend of mine, was my guide.

Within the tall walls is a spacious courtyard (keep off the grass!). The chalk paintings on the walls are reminiscent of the feudal days. The chapel is old and solemn, and is where the martyrs of World War II are remembered.



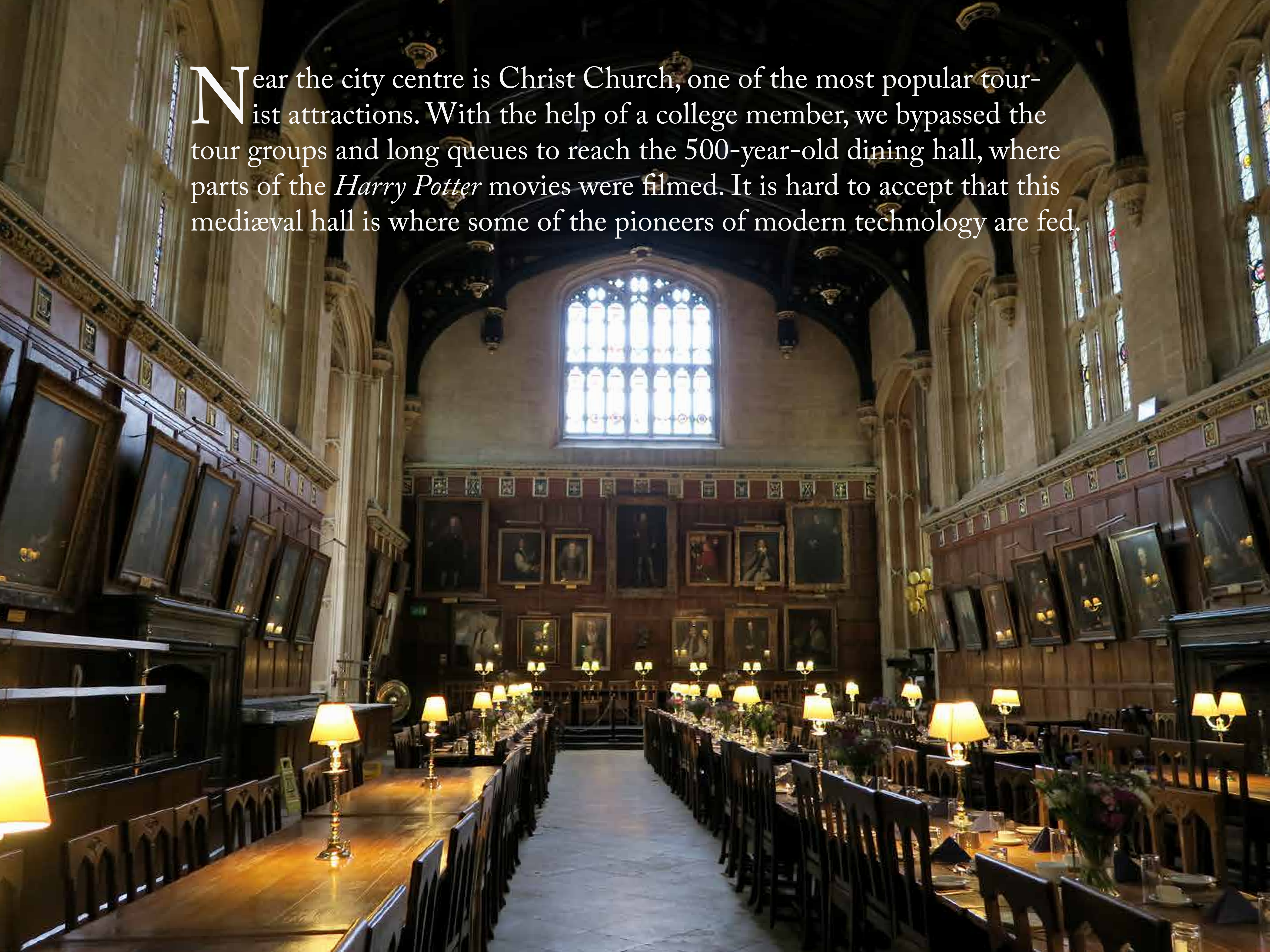
Great local pastimes include "punting"—pushing a boat using a long pole. The canals cut through Oxford, and offered a quiet area away from the dense and busy city centre.

Punting required skills and stamina, but to see ducklings and swans coasting under the shades of the willow trees made the effort worthwhile.

I had the honour to attend two piano recitals by the local pianist Jack Gibbons, featuring Chopin and Liszt, two of my favourite composers. The recitals were held in a fully packed theatre, where I spent a great musical evening before "hitting the pub".

After much reading of the background material, I could finally contribute to the project. I would study the conditions for twinning in a solid crystal, as it undergoes transition from the austenite phase to the martensite phase. This is applicable to a class of peculiar materials called shape-memory alloys.

The problem is the general version of a simpler problem that is already solved. The difficulty lies in computing the extremely long expressions (hundreds of terms) obtained from certain eigenvalue equations.



Near the city centre is Christ Church, one of the most popular tourist attractions. With the help of a college member, we bypassed the tour groups and long queues to reach the 500-year-old dining hall, where parts of the *Harry Potter* movies were filmed. It is hard to accept that this mediæval hall is where some of the pioneers of modern technology are fed.

Christ Church owns a large meadow surrounded by gravel paths and meandering streams. In the English drizzle, the meadow was especially peaceful. From a distance, one could see the hay bales and the grazing cows, which seemed oblivious to the tourists around them.

Some of the summer research students joined me on a day trip to Bath, Somerset. The Romans had built a temple over a natural mineral spring there, which, to this day, continues to gush out water that tastes of iron and sulphur.

In the afternoon, we rented some bicycles and travelled towards Bristol until the rain had stopped us.

We enjoyed cycling so much that we decided to do it again. This time, we cycled on a fine day from Oxford to Woodstock. There, we visited Blenheim Palace, the birthplace of Sir Winston Churchill. Along the path ran a canal, where boats slowly travelled from lock to lock.

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t_] := Module[{R = QRDecomposition[mat][[2]]},
 Sign[R[[i, i]]] R[[i]], {i, 1, Length[mat]}]

 list of convex combinations on a valid rank-1 line, as uniqu
, b_, λ1_, μ1_, λ2_, μ2_, step_] := Module[{mat1, mat2, lam, inbe
 N[U1 + λ1 a12.Transpose[n12] + λ2 pair[p][[2]][λ1].Transpose[pai
 N[U1 + μ1 a12.Transpose[n12] + μ2 pair[p][[4]][μ1].Transpose[pai
N[laminate[mat2, mat1]]; (* Put mat1 last to preserve the U1+λ
een = Flatten[Table[mat1 + λ3 lam[[i]], {i, 1, 2}, {λ3, 0, 1, step

_, b_, λ1_, μ1_, λ2_, μ2_] := Module[{mat1, mat2, lam},
 N[U1 + λ1 a12.Transpose[n12] + λ2 pair[p][[2]][λ1].Transpose[pai
 N[U1 + μ1 a12.Transpose[n12] + μ2 pair[p][[4]][μ1].Transpose[pai
N[laminate[mat2, mat1]](* Put mat1 last to preserve the U1+λ a
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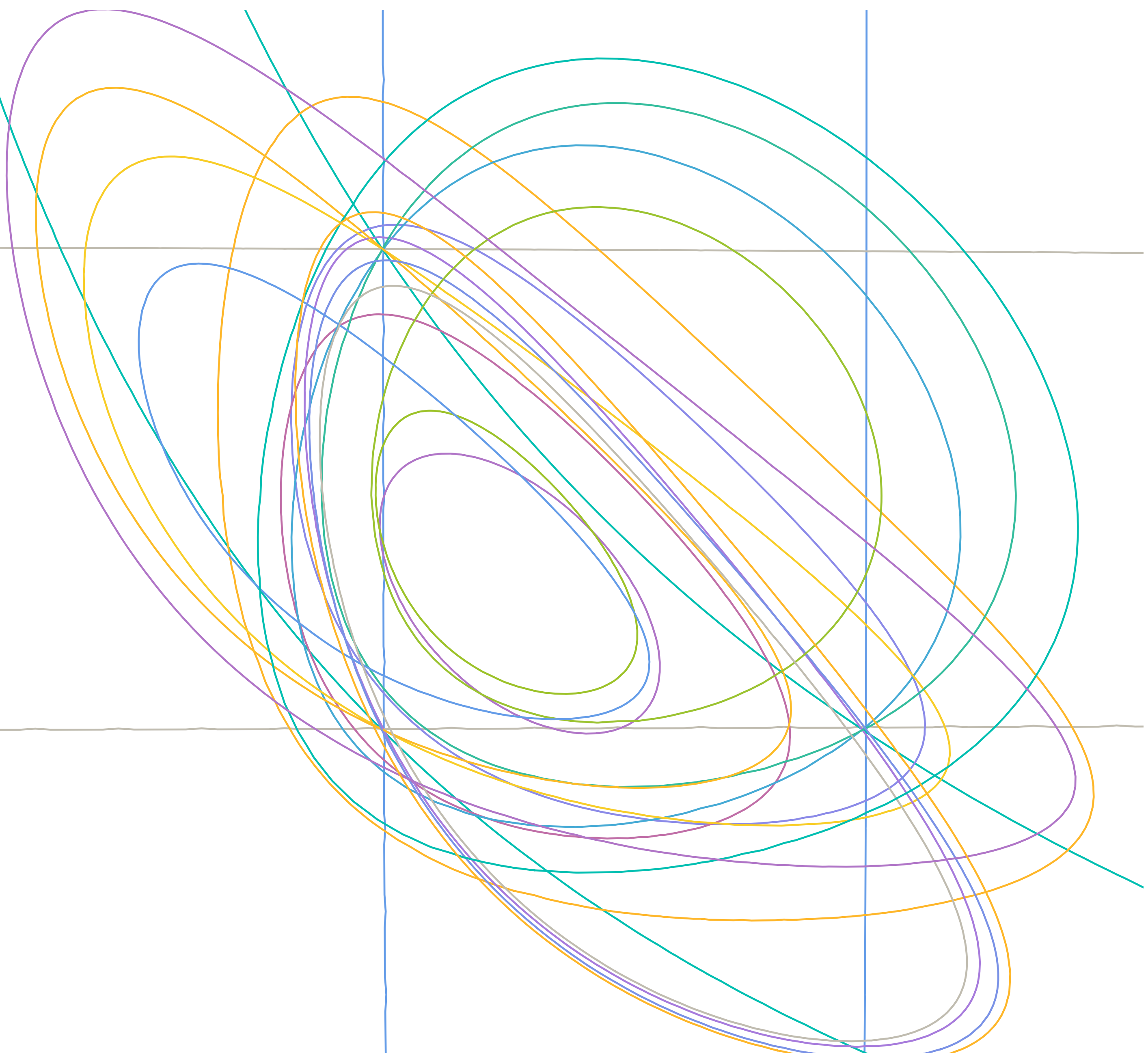
The matrix computation was indeed arduous. Due to the complexity of the equations, we employed numerical methods using Mathematica to obtain partial results. Some codes that I wrote had to run overnight, but the results looked promising.



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{{{0.61, 0., 0.0087482}, {0, 0.99, 0.005}, {0, 0, 0.9
 {{0.61, 0.01, 0.00671711}, {0, 0.99, 0.}, {0, 0, 0.9
 {{0.99, 0., 0.00633857}, {0, 1., 0.}, {0, 0, 0.60606

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Histogram[#[[3, 3]] & /@ conditionedlist]
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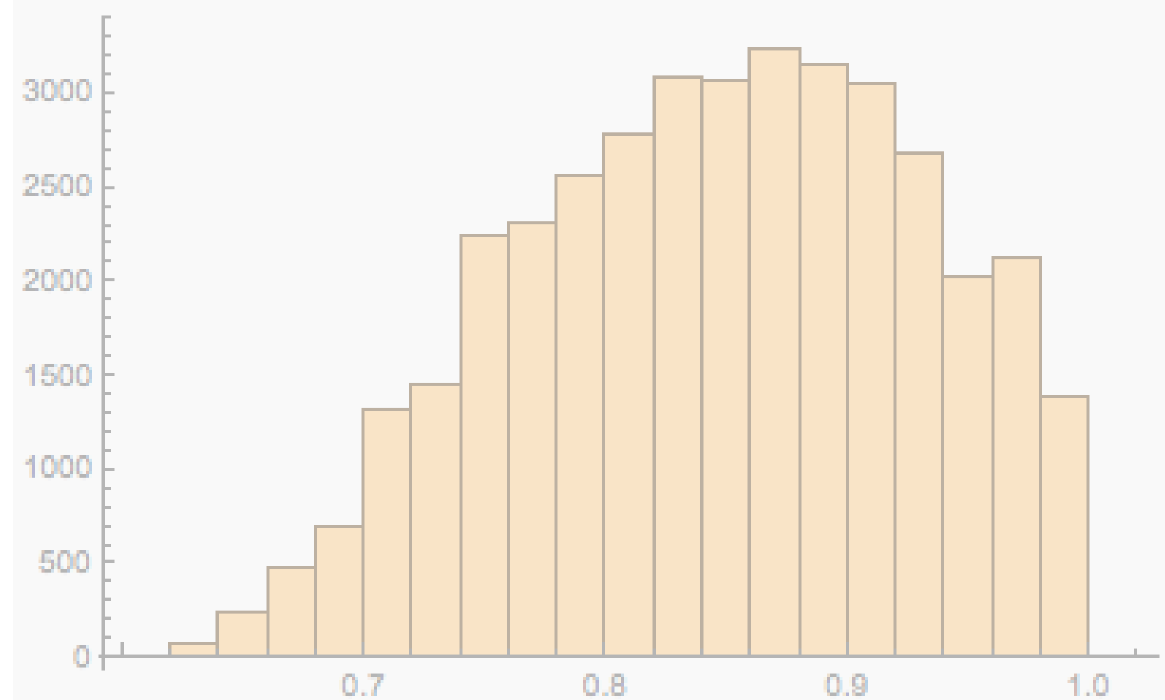


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QRpicklistsort11 = SortBy[QRpicklist, #[[1, 1]] &];
conditionedlistsort11 = SortBy[conditionedlist, #[[1,
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To conclude our stay in England, Thomas and I spent a weekend in London. We toured Westminster Palace and went on the London Eye. By pure coincidence, we encountered a royal parade just outside the Palace commemorating Victory over Japan Day.

London is a colossal city, a perfect mix of cultural heritage and modernity. With its red double-decker busses, red telephone booths, and black Hackney carriages, London is immediately recognisable.

After some fish and chips, a visit to the Natural History Museum, and an evening stroll along Thames, we were ready to return to the quiet town of Oxford.

After seven weeks of advanced mathematics, local cuisine, and the unspoilt countryside, I found this experience both intellectually challenging and physically relaxing. Even for such a brief stay, there was a sense of privilege being part of this old and charming town. I will without a doubt recommend Oxford as a destination of academic pursuit and leisurely travel.


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