Program of the Oral Qualification Examination on the topic of "Stochastic Analysis"

For students seeking the degree of
Doctor of Philosophy in Mathematical Sciences
and intending to concentrate in Mathematical Finance or Probability

March, 2009

1. Construction of stochastic processes.
   (a) The Kolmogorov extension theorem for product spaces.
   (b) Existence of continuous modifications: the Kolmogorov-Čentsov theorem.
2. Martingales, semimartingales and stopping times.
   (a) The optimal sampling theorem. Localization.
   (b) Existence of right-continuous with left limits modifications.
   (c) The Doob inequalities.
   (d) The Doob-Meyer decomposition.
   (e) Almost sure convergence of submartingales.
3. Brownian motion.
   (a) Martingale characterization.
   (b) Construction via the Kolmogorov-Čentsov theorem.
   (c) The invariance principle (convergence of random walk to Brownian motion).
   (d) The law of iterated logarithm.
4. Stochastic integration with respect to a Brownian motion.
   (a) Construction of stochastic integral.
   (b) Ito's formula.

(c) The Burkholder-Davis-Gundy inequalities.

(d) The integral representation theorem.

5. The Girsanov Theorem.

   (a) The Novikov condition.

6. Local time of Brownian motion.

   (a) The Tanaka formula.

   (b) The Trotter existence theorem.

7. Markov property and strong Markov property for Ito diffusions.

   (a) Generator and characteristic operator.

   (b) Dynkin's formula.

8. Stochastic differential equations.

   (a) Strong solutions. Existence and uniqueness.

   (b) Weak solutions. Existence and uniqueness.

   (c) Probabilistic representations for the solutions to Dirichlet and Cauchy problems.

   (d) Linear stochastic differential equations.

   (e) Ornstein-Uhlenbeck process.

   (f) Brownian bridge.

9. Stochastic control.

   (a) Optimal stopping problem. Connection with variational inequalities.

   (b) The Hamilton-Jacobi-Bellman equation.

References

[1] Ioannis Karatzas and Steven E. Shreve. *Brownian motion and stochastic calculus*, volume 113 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 1991.

[2] Bernt Øksendal. *Stochastic differential equations*. Universitext. Springer-Verlag, Berlin, sixth edition, 2003. An introduction with applications.

[3] L. C. G. Rogers and David Williams. *Diffusions, Markov processes, and martingales. Vol. 1.* Cambridge Mathematical Library. Cambridge University Press, Cambridge, 2000. Foundations, Reprint of the second (1994) edition.

[4] L. C. G. Rogers and David Williams. *Diffusions, Markov processes, and martingales. Vol. 2.* Cambridge Mathematical Library. Cambridge University Press, Cambridge, 2000. Itô calculus, Reprint of the second (1994) edition.