CNA Working Group, Spring 2009
Rigidity Properties of Thin Structures
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Elastic thin objects (such as rods, plates, shells) of various geometries are ubiquitous in the physical world and precise understanding of laws governing their equilibria has many applications. The purpose of this working group is to study various aspects of thin structures together with their geometric properties (such as rigidity) which influence their deformation under forces and/or boundary conditions. Other significant problems, such as the relationship of the discussed theories with the 3d theory of nonlinear elasticity, and rigorous derivation of some mechanical principles known to engineers, will be considered.

As a starting point, we shall discuss the following topics:

1. The geometric rigidity estimate, which paved the way for rigorous derivation of various plate theories through methods of Calculus of Variations. [presented in class of prof. Fonseca]

   G. Friesecke, R. James and S. Müller, *A theorem on geometric rigidity and the derivation of nonlinear plate theory from three dimensional elasticity*, Comm. Pure. Appl. Math., **55** (2002), 1461–1506.

2. A self-contained proof of the classical Liouville theorem for conformal mappings, whose quantitative version is the rigidity theorem by Faraco and Zhong below. [Pawel Konieczny]

   T. Iwaniec and G. Martin, *The Liouville theorem*, Analysis and topology, 339–361, World Sci. Publ., River Edge, NJ (1998).

3. The geometric rigidity estimate for conformal maps. The relation of this result to the conformal Liouville theorem is the same as the Friesecke James Muller estimate to the Liouville theorem (both results are at the nonlinear level, their linearizations are, respectively, the conformal Korn (see also paper by Sergio Dain in Calc Var 2007) and the standard Korn's inequality. [Marco Barchiesi]

   D. Faraco and X. Zhong, *Geometric rigidity of conformal matrices*, Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) **4** (2005), no. 4, 557–585.

4. The rigorous derivation of the plate model relating to the elastic energy of deformations (per unit thickness h) scaling like h^β with $\beta \in (0, 5/3)$. The limiting case $\beta = 5/3$ is conjectured to be related to paper crumpling. [Filippo Cagnetti]

   S. Conti and F. Maggi, *Confining thin elastic sheets and folding paper*, Arch. Ration. Mech. Anal. **187** (2008), no. 1, 1–48.

   One can also look at: S.C. Venkataramani, *Lower bounds for the energy in a crumpled elastic sheet—a minimal ridge*, Nonlinearity **17** (2004), no. 1, 301–312.

5. Introduction of the von Kármán theory ($\beta = 4$) in the general setting of static elasticity of thin shells, around a mid-surface of arbitrary geometry. The paper exposes which intrinsic geometrical properties of 2d surfaces play role in the elastic response of a shell. [Milena Chermisi]

   M. Lewicka, M.G. Mora, R. Pakzad, *Shell theories arising as low energy Γ-limit of 3d nonlinear elasticity*, http://arxiv.org/abs/0803.0358

6. Other approaches to justification of elasticity theories of thin shells/plates, using center manifold theorem. [Stefan Kroemer]

   A. Mielke, *Saint-Venant's problem and semi-inverse solutions in nonlinear elasticity*, Arch. Rational Mech. Anal. **102** (1988), no. 3, 205–229.

   Its Corrigendum: A. Mielke, *Corrigendum to: "Saint-Venant's problem and semi-inverse solutions in nonlinear elasticity" [Arch. Rational Mech. Anal. 102 (1988), no. 3, 205–229]*, Arch. Rational Mech. Anal. **110** (1990), no. 4, 351–352.

7. Another justification of the von Kármán theory, through the implicit function theorem. [?]

   R. Monneau, *Justification of nonlinear Kirchhoff-Love theory of plates as the application of a new singular inverse method*, Arch. Rational Mech. Anal. **169** (2003), 1–34.