Periodic Frenkel-Kontorova Model

We consider a double-sided infinite sequence of particles, connected by springs and placed in a periodic potential. The state of the system is given by a sequence $\{x_i\}_{i\in\mathbb{Z}} \in \mathbb{R}^{\mathbb{Z}}$. The formal energy of the system is

$$\mathscr{E}(\{x_i\}_{i\in\mathbb{Z}}) = \sum_{n\in\mathbb{Z}} \frac{1}{2}(x_{n+1} - x_n - a)^2 - V(x_n)$$

where the potential V is choosen to be a periodic function.

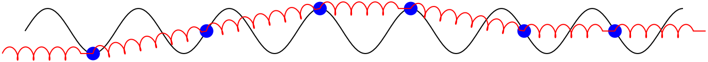

Figure: Frenkel-Kontorova Model

Quasi-Periodic FK Model

We will consider Frenkel-Kontorova models with quasi-periodic potentials. The formal energy of the system is

$$\mathscr{E}(\{x_i\}_{i\in\mathbb{Z}}) = \sum_{n\in\mathbb{Z}} \frac{1}{2}(x_{n+1} - x_n - a)^2 - V(x_n)$$

where the potential V is choosen to be a quasi-periodic function with irrational frequency α

$$V(\theta) = \hat{V}(\theta\alpha)$$

where $\alpha \in \mathbb{R}^d$ is an irrational vector.
$\left(k \cdot \alpha \notin \mathbb{Z}, \quad \forall k \in \mathbb{Z}^d - \{0\}\right)$. $\hat{V}$ is a function from $\mathbb{T}^d$ to $\mathbb{R}$.

Example

$V(\theta) = \sin(2\pi\theta) + \frac{1}{2}\sin(2\pi\theta \cdot \sqrt{2})$,
where $\hat{V}(\theta_1, \theta_2) = \sin(2\pi\theta_1) + \frac{1}{2}\sin(2\pi\theta_2)$, $\alpha = (1, \sqrt{2})$.