Compactness and Structural Stability of Nonlinear Flows

Augusto Visintin (Trento, Italy) — Dallas and Pittsburgh, October 2017

After the seminal works [1],[2] of Brezis, Ekeland, Nayroles and Fitzpatrick, maximal monotone operators $\alpha : V \to \mathcal{P}(V')$ (V being a Banach space) and flows of the form

$$\frac{du}{dt} + \alpha(u) \ni h \quad \text{in } V', \text{ a.e. in }]0,T[, \qquad u(0) = u^0 \tag{1}$$

can be formulated as a minimization principle, even if α is not a subdifferential.

On this basis, De Giorgi's notion of Γ-convergence may be applied to the analysis of monotone inclusions. Compactness and structural stability of the Cauchy problem are then studied, with respect to arbitrary variations not only of the datum $h \in L^2(0,T;V')$, but also of the operator α. This suggests the use of an exotic nonlinear topology of weak type, see [4]–[7].

The operator α may also be assumed to be a multivalued pseudo-monotone operator, e.g.:

$$\alpha(u) = -\nabla \cdot \vec{\gamma}(u, \nabla u) \qquad \forall u \in W_0^{1,p}(\Omega),$$

with $\vec{\gamma}$ lower semicontinuous (as a multivalued operator) w.r..t. the first argument, and maximal monotone w.r..t. the second one.

These results can be extended in several directions, and can be applied to nonlinear either stationary of evolutionary PDEs. In particular, one can deal with doubly-nonlinear parabolic inclusions of the form

$$D_t \partial\varphi(u) + \alpha(u) \ni h \qquad \text{or} \qquad \alpha(D_t u) + \partial\varphi(u) \ni h, \tag{2}$$

with α a above and φ convex and lower semicontinuous.

These recent results rest upon a novel notion of *evolutionary* Γ*-convergence* of weak-type [8].

This research is surveyed in [6].

References

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