Overview: Nature is full of energy-driven patterns. Some represent local or global minimizers of a suitable free energy. Others are self-organized transients produced by energy-dissipating dynamics. Simulation can demonstrate the adequacy of a model, but it rarely explains "why" a pattern forms. Nonlinear PDE and the calculus of variations can sometimes provide a more global understanding. I’ll give four independent lectures on problems of this type, followed by a fifth lecture that’s a bit different.

Note: references on which I’m an author can be downloaded as pdf files from www.math.nyu.edu/faculty/kohn.

Lecture 1: Bounds on coarsening rates. Some energy-driven systems develop interesting patterns transiently (as they evolve) rather than in steady state (at local minima). An example is the coarsening of a complex initial state under motion by surface diffusion. In this setting (and many others), the “local length scale” increases with time, often with an exponent that can be guessed by dimensional analysis. I’ll introduce this phenomenon, then discuss a scheme introduced with F. Otto a few years ago for proving an upper bound on the coarsening rate, focusing on one of the earliest applications: “motion by surface diffusion.”


2. R. Kohn and X. Yan, Upper bounds on the coarsening rate for an epitaxial growth model, Comm. Pure App. Math. 56 (2003) 1549-1564 (this paper’s introduction is shorter and more focused than the CMP paper)

3. R. Kohn and X. Yan, Coarsening rates for models of multicomponent phase separation, Interfaces and Free Boundaries 6 (2004) 135-149 (again, this paper’s introduction is shorter and more focused than the CMP paper)


**Lectures 2 and 3: The internal structure of a cross-tie wall.** The cross-tie wall is a particular type of domain wall that forms in soft, thin ferromagnetic films. I’ll explain its structure by identifying an associated variational problem, then showing that the pattern we see achieves its minimum. Central issues include (a) the relation between sharp-interface and diffuse-interface models, and (b) use of suitable “entropies” to prove lower bounds on the energy of a boundary value problem. In exploring these issues we’ll discuss the Modica-Mortola problem and the Aviles-Giga problem as well as the Alouges-Rivi`ere-Serfaty picture of a cross-tie wall.


3. F. Alouges, T. Rivi`ere, S. Serfaty, *Néel and cross-tie wall energies for planar magnetic configurations*, ESAIM:COCV 8 (2002) 31-68 (more general than my treatment; their formulation has diffuse rather than sharp walls)


**Lecture 4: The sharp-interface limit of action minimization.** Energy-driven systems typically achieve local not global minima. Thermal fluctuations lead to switching from one local minimum to another. The action functional identifies the rate and most likely pathway of switching. I’ll introduce this topic, then consider the sharp-interface limit of action minimization for the Modica-Mortola functional, drawing on recent joint work with F. Otto, Y. Tonegawa, E. Vanden-Eijnden, and M. Westdickenberg.


in “early-view” (recent progress on numerical methods for finding action-minimizing paths in non-gradient systems)


**Lecture 5: Cloaking by change of variables.** We say a region of space is “cloaked” with respect to electromagnetic measurements if its contents – and even the existence of the cloak – are inaccessible to such measurements. One recent proposal for achieving cloaking takes advantage of the coordinate-invariance of Maxwell’s equations. I’ll explain this scheme, including its mathematical basis and its apparent limitations, drawing on recent work with Onofrei, Shen, Vogelius, and Weinstein.

1. R. Kohn, H. Shen, M. Vogelius, and M. Weinstein, *Cloaking via change of variables in electric impedance tomography*, Inverse Problems 24 (2008) 015016 (this paper includes a long, expository introduction and is a lot like my lecture)


5. R. Kohn, D. Onofrei, M. Vogelius, and M. Weinstein, in preparation (this work discusses the design of a change-of-variable-based “near-cloak” in the finite-frequency setting, for a system described by Helmholtz’s equation).