Asymptotics for Exit Problem and Principal Eigenvalue for a Class of Non-Local Elliptic Operators Related to Diffusion Processes with Random Jumps

Ross Pinsky

Technion–Israel Institute of Technology, Haifa

Let $D \subset R^d$ be a bounded domain and denote by $\mathcal{P}(D)$ the space of probability measures on D. Let

$$L = \frac{1}{2}\nabla \cdot a\nabla + b\nabla$$

be a second order elliptic operator on D. Let $\mu \in \mathcal{P}(D)$ and $\delta > 0$. Consider a Markov process $X(t)$ in D which performs diffusion in D generated by the operator δL and is stopped at the boundary, and which while running, jumps instantaneously, according to an exponential clock with spatially dependent intensity $V > 0$, to a new point, according to the distribution μ. Let $P_x^{\delta,\mu}(\cdot)$ denote probabilities for the process starting from x. The Markov process is generated by the operator $L_{\delta,\mu,V}$ defined by

$$L_{\delta,\mu,V}\phi \equiv \delta L\phi + V(\int_D \phi \, d\mu - \phi).$$

Let $\lambda_0(\delta,\mu) > 0$ denote the principal eigenvalue for $-L_{\delta,\mu,V}$. Let $\tau_D = \inf\{t \geq 0 : X(t) \in \partial D\}$ denote the first exit time of the process from D. The well-known connection between the principal eigenvalue and τ_D is given by

$$\lim_{t\to\infty} \frac{1}{t} \log P_x^{\delta,\mu}(\tau_D > t) = -\lambda_0(\delta,\mu).$$

Let $\nu_{\delta,\mu,x}(\cdot) = P_x^{\delta,\mu}(X(\tau_D) \in \cdot)$ denote the exit measure for the process starting at x. As $\delta \to 0$, $\nu_{\delta,\mu,x}$ converges weakly to a limiting measure ν_μ, independent of x. We study the asymptotic behavior of $\lambda_0(\delta,\mu)$ as $\delta \to 0$ and the behavior of the limiting exit measure ν_μ.