A Two-Person Game for Pricing Convertible Bonds

A firm issues a convertible bond. At each subsequent time, the bondholder must decide whether to keep the bond, thereby collecting coupons, or to convert it to stock. The bondholder wishes to choose a conversion strategy to maximize the bond value. Subject to some restrictions, a convertible bond can be called by the issuing firm, which presumably acts to maximize equity value and thus to minimize the bond value. This creates a two-person game, and we model the bond price as the value of this game. We show, however, that under our standing assumption (dividends are paid at a lower rate than the money market rate) this game reduces to one of two optimal stopping problems, and the relevant stopping problem can be determined a priori, i.e., without first solving the convertible bond pricing problem.

Because of dividend payments, the partial differential equation describing the pricing function becomes nonlinear. This means that our analysis involves a fixed point problem. We also prove that for large time to maturity the value of the convertible bond approaches the value of the perpetual convertible bond.

The presentation is based on joint work with Steven E. Shreve.