

Gittins Theory, Index and Theorem in a General Form

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The talk will consist of two parts. The goal of the main part is to present the renowned Gittins Theory, Index and Theorem in the most general, nonstationary form, at least for the discrete time case, and to explain the idea of the proof of the Theorem. The main point of the generalization is that instead of the multi-armed bandit setting with independent "arms", where each arms is some stationary probability models, we consider nonstationary evolving environment with a stochastic structure of this evolution. This approach that was initiated in [4], and was continued more recently in [1] and [3].

This general setting can be described as follows. Let us imagine that you - a Reader, a Decision Maker (DM) are in a magic library. At an initial moment some "pages" are "open", i.e available for "reading". At this moment, as well as at each moment of discrete time you can select one page for reading. If page e is chosen then you receive a reward $r(e)$. As a result of reading, with some probability, which depends on the chosen page, the process of reading is terminated, if not then the chosen page is closed forever and with some probability a set (possibly empty) of new pages is added at the next moment to the set of already open pages. You know all the rewards and all the parameters of stochastic evolution. The goal is to select an optimal strategy of reading to maximize the expected total reward.

This problem has a remarkable answer obtained by Gittins for the stationary situation. For each page there is an index $\alpha(e)$, based only on the rewards and stochastic structure of pages that "follow" e , and an optimal strategy is to use each time a page with the highest value of α among all open.

The second, smaller part of the talk is based on a personal experience of the author as an amateurish investor. In this part I will address the following question. In any source which deals with practical investment you will find that the most popular term is the "resistance level", and its counterpart, the support level, like: "The DJIA is still staring up at resistance in the 10,450 region, where the blue-chip barometer ...The DJIA is now trading back below resistance in the 10,500 area, with potential support just below...

The resistance level is described as "the price at which a stock or market can trade, but not exceed, for a certain period of time".

I have not seen such terms in serious literature in mathematical finance. Do such things really exist ? I will discuss a sketch of the model which can explain the existence

of these two levels. Some though weak resemblance with the Gittins index behavior can be noticed in this model.

References

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