Small-time Asymptotics of Options Prices Under Lévy-based Models

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The short-time asymptotic behavior of options prices has received much attention in recent years. In this talk, we study the small-time behavior of call-option prices $E(S_t - K)^+$ under a variety of models with Lévy jumps.

In the out-of-the-money (OTM) and in-the-money (ITM) cases, we consider a stochastic volatility model with Lévy jumps for a log-return process $Z = (Z_t)_{t \geq 0}$ of the form $Z = U + X$, where $U = (U_t)_{t \geq 0}$ is a classical stochastic volatility process and $X = (X_t)_{t \geq 0}$ is an independent pure-jump Lévy process with absolute continuous Lévy measure. In that setting, small-time expansions, of arbitrary polynomial order, in time-$t$, are obtained for OTM and ITM call-option prices, assuming smoothness conditions on the Lévy density away from the origin and a small-time large deviation principle on $U$. As a consequence, the small-time asymptotic behavior of the corresponding Black-Scholes implied volatility is derived.

In the at-the-money (ATM) case, a novel second-order approximation is obtained for the CGMY Lévy model and then extended to a model with an additional independent Brownian component. The third-order asymptotic behavior of the option prices as well as the asymptotic behavior of the corresponding Black-Scholes implied volatility is also addressed.