Market Breakdown and Indeterminacy under Model Uncertainty

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Outline

1. Uncertainty versus Risk
2. Model
3. Efficiency and Equilibria in Bewley Economies
4. Market Breakdown (No Trade Equilibria) and Indeterminacy
Uncertainty versus Risk

- **Roulette versus Horse Races**
- objective probability versus no probabilities, just uncertain outcomes
- \((\Omega, \mathcal{F}, P)\) probability space versus \((S, \mathcal{S})\) measurable space, 
  \(X : (S, \mathcal{S}) \rightarrow \mathbb{R}\)

Savage, Anscombe–Aumann
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- events $A, B \subseteq S$,
  $1_A \succ 1_B \iff \text{"A" is more probable than "B"} \iff P(A) > P(B)$ for some subjective probability $P$
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Uncertainty II: Ellsberg

Asking for exact subjective probabilities too demanding in many real–world decisions.

Example

Imagine a baseball fan. He has the choice between two bets.

Situation 1:
- SF Giants win the World Series
- Arminia Bielefeld wird Pokalsieger 2013.

Situation 2:
- SF Giants do not win the World Series
- Arminia Bielefeld wird nicht Pokalsieger 2013.

It is perfectly rational to go for the first bet in both cases; but this would contradict the additivity of probability.
Knight (1921): many economic decisions are of a one-shot nature and one cannot presume probabilities

- Probability fairly well known for
  - Car Insurance
  - Life Insurance (Mortality Risk)
  - "IBM"

- Probability less clear for
  - market entry
  - patents
  - Startups
  - Rating 'BB'
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Uncertainty IV: Two Formal Models

As “P” is not exactly known, work with a whole class of probability measures $\mathcal{P}$, (Huber, 1982, Robust Statistics)

**Bewley’s Model: uniform multiple prior approach**

- incomplete expected utility
- $X \preceq Y \iff \forall P \in \mathcal{P} \ E^P u(X) \leq E^P u(Y)$
- plus inertia
- agents move from status quo $\omega^i$ to $x^i$ iff $E^P u(x^i) > E^P u(\omega^i)$ for all priors $P \in \mathcal{P}^i$
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Our paper

- Complete General Equilibrium Analysis of Dynamic Economies with Incomplete Expected Utility Preferences and Inertia
- Case Study: Market Breakdown and Inertia, Indeterminacy
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Literature

- Bewley’s papers on multiple priors
  - Cowles Discussion Papers 1986, 87, 89
  - Part II in Hildenbrand’s Festschrift
- Sargent, Robustness (textbook)
Model: Bewley and Savage Economies

Definition

1. B–economy = Standard dynamic exchange economy under uncertainty, except for incomplete multiple–prior preferences given by a set of priors $\mathcal{P}^i$ for agent $i$.

2. Fix priors $Q^i \in \mathcal{P}^i$.
   S–economy with priors $Q = (Q^1, Q^2, \ldots, Q^I) =$ complete preferences, and possibly heterogeneous priors $Q = (Q^1, Q^2, \ldots, Q^I)$.

3. S for Savage, not a risk economy!
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Uncertainty versus Risk Model Efficiency Breakdown

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Model ctd.

- $I$ agents with multiple priors $\mathcal{P}^i$
  - priors admit densities with respect to a reference measure $P^0$
  - agents agree on null sets
  - for $Q^i \in \mathcal{P}^i$, we denote the density process by $q^i_t$
- Consumption plans $c^i = (c^i_t(\omega))$ for $t = 0, 1, \ldots, T$
- agent $i$ weakly prefers $c^i$ over $d^i$ iff
  
  $$E^Q \sum_{t=0}^{T} u^i(t, c^i_t) \geq E^Q \sum_{t=0}^{T} u^i(t, d^i_t)$$

- $u^i$ nice period utility function
- endowments $\omega^i = (\omega^i_t(\omega))$ are strictly positive
- focus on interior allocations
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An allocation $c$ consists of consumption plans $c^i = (c^i_t(\omega))$, $i = 1, \ldots, I$

- $c$ is feasible if $\sum_i c^i = \sum_i \omega^i$

- $c$ is efficient if there is no other feasible allocation $d$ such that $d^i \succ^i c^i$ for all agents $i$
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Efficiency in Savage Economies

Fix priors $Q = (Q^1, Q^2, \ldots, Q^I)$

A feasible interior allocation $c = (c^1, c^2, \ldots, c^I)$ is efficient in the S–economy with priors $Q = (Q^1, Q^2, \ldots, Q^I)$ iff the marginal rates of substitution of all agents coincide, i.e.

$$MRS^i_t(Q^i) = \frac{u^i_c(t, c^i_t) q^i_t}{u^i_c(c^i_0)} = \frac{u^j_c(t, c^j_t) q^j_t}{u^j_c(c^j_0)} = MRS^j_t(Q^j)$$
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Efficiency in Bewley economies

More or less trivial:

Lemma

if c is efficient in some S–economy with priors Q, then c is efficient in the B–economy.

Challenge: the converse!
Efficiency in Bewley economies

MRS=Risk–Adjusted Prior + Subjective Interest Rate

- Every MRS can be written as

\[ MRS_t^i = \frac{u_c^i(t, c_t^i)q_t^i}{u_c^i(c_0^i)} = M_t^i \exp \left( -\sum_{s=1}^{t} r_s^i \right) \]

for a martingale \( M^i = M^i(Q^i) \) with expectation 1 and a subjective interest rate \( r^i \)

- Interest rate is predictable
- Decomposition is unique (Multiplicative Doob Decomposition)
- \( M^i \) density process of a new measure, the risk–adjusted prior or equivalent martingale measure
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$$MRS_t^i = \frac{u_t^i(t, c_t^i) q_t^i}{u_0^i(c_0^i)} = M_t^i \exp \left( - \sum_{s=1}^{t} r_s^i \right)$$

for a martingale $M_t^i = M_t^i(Q_t^i)$ with expectation 1 and a subjective interest rate $r_i$

- Interest rate is predictable

- Decomposition is unique (Multiplicative Doob Decomposition)

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Efficiency in Bewley economies

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For agent $i$, let

$$\mathcal{MRP}^i = \{ MRS^i(Q^i) : Q^i \in \mathcal{P}^i \}$$

be the set of agent $i$’s marginal rates of substitution. Let

$$\mathcal{M}^i = \{ M^i(Q^i) : Q^i \in \mathcal{P}^i \}$$

be the set of agent $i$’s equivalent martingale measures.
An interior allocation $c$ is efficient in the Bewley economy if and only if one of the following conditions holds true:

1. the agents’ share a common marginal rate of substitution, \[ \cap_i MRS^i \neq \emptyset \]

2. the agents share a risk–adjusted prior and for a common risk–adjusted prior $Q$ all individual interest rates are equal, i.e.
   \[ r^i(Q, c^i)_t = r^j(Q, c^j)_t \]
   for all $i, j = 1, \ldots, I$ and $t = 0, \ldots, T$,

3. for some selection of priors $Q^i \in \mathcal{P}^i, i = 1, \ldots, I$, $c$ is efficient in the Savage economy with priors $Q = (Q^1, \ldots, Q^I)$. 

Efficiency in Bewley economies

Theorem

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Samet’s Theorem

Theorem (Samet, Games and Economic Behavior, 1998)

Let $K_1, \ldots, K_n$ be convex, closed, nonempty subsets of the simplex $\Delta^m$ in $\mathbb{R}^m$.

$\bigcap K_i = \emptyset$ iff there are $f_1, \ldots, f_n \in \mathbb{R}^m$ such that $\sum f_i = 0$, and $f_i \cdot x_i > 0$ for each $x_i \in K_i, i = 1, \ldots, n$. 

Separation Theorem

Samet’s Theorem
Samet’s Theorem for $L^\infty$

**Theorem**

Let $(S, \mathcal{I}, P)$ be a probability space. Let $(K_i)_{i=1,...,n}$ be nonempty, convex, and $\sigma(L^1(S, \mathcal{I}, P), L^\infty(S, \mathcal{I}, P))$-compact subsets of $\Delta = \{D \in L^1_+(S, \mathcal{I}, P) : E D = 1\}$.

Then $\bigcap K_i = \emptyset$ if and only if there exists $g_i \in L^\infty(S, \mathcal{I}, P)$ with $\sum g_i = 0$ such that $\int g_i x_i dP > 0$ for all $x_i \in K_i$, $i = 1, \ldots, n$. 
Equilibria in Bewley economies

An (Arrow–Debreu) equilibrium consists of a (state) price process \( p = (p_t) \) (adapted), and a feasible allocation \( c \) such that for each agent \( i \)

\[
d^i \succ^i c^i \implies E^{P_0} \sum p_t d^i_t > E^{P_0} \sum p_t c^i_t
\]

Corollary

Any interior equilibrium \( (p^*, c^*) \) of the Bewley economy is an interior equilibrium for some Savage economy with priors \( Q^i \in \mathcal{P}^i, i = 1, \ldots, I \) and vice versa.

Remark

- Huge number of equilibria if uncertainty is nontrivial
- Indeterminacy (compare Rigotti–Shannon)
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Equilibria with Inertia: Existence

- Inertia: agents choose $c^i \neq \omega^i$ only if they strictly prefer $c^i$ over $\omega^i$ under all $P \in \mathcal{P}^i$
- Big reduction of number of equilibria
- New Idea: introduce a certain class of variational preferences (Maccheroni, Marinacci, Rustichini) with reference level $\omega^i$

$$V^i(x) = \min_{Q \in \mathcal{P}^i} E^Q \left( (U^i(x) - U^i(\omega^i)) \right)$$  (1)

Theorem (Existence)

Any equilibrium of an economy with complete variational preferences (1) is an equilibrium with inertia (in the B–economy). In particular, equilibria with inertia exist.

Technical Remark

Such variational preferences are Mackey–continuous.
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No Trade and Indeterminacy

CAPM–like Model with Uncertainty

- No aggregate uncertainty
- Individual endowments depend on risky source (distribution known) and uncertain source (distribution unknown)
- Agents agree that risk and uncertainty are independent
- Constant absolute risk aversion and normal distributions incomplete mean–variance model
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Results in a Nutshell

- Risk is completely insured, and thus there is always trade,
- Uncertainty is fully insured if ambiguity is small,
- When ambiguity exceeds a certain critical level, the full insurance allocation (where also the uncertain part of the endowment is fully insured) is not an equilibrium with inertia,
- And there is an equilibrium with inertia where subjective uncertainty is not insured at all,
- Inertia and uncertainty lead to no trade,
- Finally, although inertia is a strong equilibrium refinement, we nevertheless have indeterminacy of equilibria with inertia.
Case Study: Details

- two agents with CARA utility, \( u^i(x) = -\exp(-x) \)
- aggregate endowment is zero
- agent 1 has endowment \( \omega_1 = R_t + U_t \)
- \( R \) is risky and \( U \) is uncertain
- \( R_t = \sum_{s=1}^{t} \varepsilon_s \), \( \varepsilon_s \sim N(0, 1) \), i.i.d.
- \( U_t = \sum_{s=1}^{t} \nu_s \), (\( \nu_t \)) independent experiments with identical ambiguity
- time–consistent dynamic model of multiple priors

\[
q_t = \exp \left( \sum_{s=1}^{t} \left( \alpha_s \nu_s - \frac{1}{2} \alpha_s^2 \right) \right)
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for some \( U \)–predictable process \( (\alpha_s) \) with values in \([-\kappa, \kappa]\)

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Textbook knowledge: with homogeneous priors and expected utility, full insurance in equilibrium

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Case Study: Full Insurance is No Equilibrium with Inertia under High Uncertainty

- Full Insurance of *both* risk and uncertainty is an equilibrium in the S–economy with homogeneous priors.
- Hence an equilibrium in the B–economy.
- But not an equilibrium with inertia if uncertainty is high.
- Intuition: every agent finds one very optimistic prior where she prefers uncertainty over insurance.
- ... positive mean.
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An equilibrium consists of an adapted, bounded price process \( (p_t) \) and a feasible allocation \( c = (c_t^i) \) such that the budget constraint

\[
E^P_0 \sum_t p_t (c_t^i - \omega_t^i) = 0
\]

is satisfied.

\((p, c)\) is an equilibrium with inertia if for each agent \( i \) we have either \( c_t^i = \omega_t^i \) or

\[
E^Q \sum_t u^i(t, c_t^i) > E^Q \sum_t u^i(t, \omega_t^i)
\]

for all \( Q \in \mathcal{P}^i \).
Equilibria

Lemma

Let \((\alpha_s), (\beta_s)\) be two \(\mathcal{F}^U\)-predictable processes with values in \([-\kappa, \kappa]\). The allocations \((c^1_1, -c^1_1)\) with

\[
\begin{align*}
c^1_t - c^1_0 &= \frac{1}{2} \left( \sum_{s=1}^{t} (\alpha_s - \beta_s) \nu_s + \frac{\beta_s^2 - \alpha_s^2}{2} \right) 
\end{align*}
\]

are efficient in the B–economy. For properly chosen \(c^1_0\) and price \(p\), they form an equilibrium.

In particular for \(|\alpha| \leq \kappa\), the allocation \((\alpha U_t, -\alpha U_t)\) with the deterministic price \(p_t = \exp\left(-\left(\rho + 1/2\alpha^2\right)t\right)\) is an equilibrium.

Remark

Indeterminacy of the interest rate in equilibrium: \(\rho + \frac{\alpha^2}{2}\) for \(\alpha \in [-\kappa, \kappa]\).
Equilibria

Lemma

Let \((\alpha_s), (\beta_s)\) be two \(\mathcal{F}^U\)-predictable processes with values in \([-\kappa, \kappa]\). The allocations \((c^1, -c^1)\) with

\[
c_t^1 - c_0^1 = \frac{1}{2} \left( \sum_{s=1}^t (\alpha_s - \beta_s) \nu_s + \frac{\beta_s^2 - \alpha_s^2}{2} \right)
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are efficient in the B–economy. For properly chosen \(c_0^1\) and price \(p\), they form an equilibrium.

In particular for \(|\alpha| \leq \kappa\), the allocation \((\alpha U_t, -\alpha U_t)\) with the deterministic price \(p_t = \exp \left( -\left(\rho + 1/2\alpha^2\right)t \right)\) is an equilibrium.

Remark

Indeterminacy of the interest rate in equilibrium: \(\rho + \frac{\alpha^2}{2}\) for \(\alpha \in [-\kappa, \kappa]\).
In every equilibrium \((p, c)\) of the B–economy, risk is fully insured in the sense that the equilibrium allocation \(c\) depends only on \(U\) not on \(R\).
Equilibria with Inertia, Full Insurance

Theorem

For sufficiently small levels of ambiguity, namely $\kappa < 1$, the full insurance allocation $c^1 = c^2 = 0$ is an equilibrium allocation with inertia.

If ambiguity is sufficiently large, $\kappa \geq 1$, the full insurance allocation $c^1 = c^2 = 0$ is not an equilibrium allocation with inertia.
Equilibria with Inertia, No Insurance of Uncertainty

Theorem

For large levels of uncertainty, $\kappa \geq 1$, there is an equilibrium with inertia in which agent 1 consumes

$$c_t^1 = U_t.$$ 

The equilibrium price is

$$p_t^* = \exp \left( - \left( \rho + \frac{1}{2} \right) t \right).$$
The equilibria with inertia are indeterminate.

For $\kappa < 1$, there exists a continuum of equilibria with inertia with allocation $(c^1, -c^1)$ and $c^1 = \varepsilon U$ for sufficiently small $\varepsilon \neq 0$.

For $\kappa \geq 1$, there exists a continuum of equilibria with inertia with allocation $(c^1, -c^1)$ and $c^1 = (1 - \varepsilon)U$ for sufficiently small $\varepsilon \neq 0$.

Intuition:
The insurance of $R$ gives a positive utility gain. This allows to trade some parts without violating the inertia constraint.
A Related Motivation: Control of Investment Banks

Regulation of Financial Markets

Regulation can be interpreted as “imposing preferences”

- **stress testing**: accept a deal only if it performs better than status quo in all tests ⇔ Bewley with inertia
- **worst–case approach**: compare the worst–case outcomes deal versus status quo and accept a deal if the worst–case outcome of the deal is better the the worst–case outcome of the status–quo

- Coherent Risk Measures (Artzner, Delbaen, Eber, Heath ⇔ Gilboa–Schmeidler
- Convex Risk Measures (F¨ ollmer, Schied, Fritelli, Giannin) ⇔ Variational Preferences
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Conclusion

1. General Equilibrium Analysis for Bewley’s Incomplete Preference Approach
2. Link to Variational Expectations
3. Samet’s Theorem for $L^\infty$
4. Link to Regulation of Financial Markets:
   - Regulation is a way to impose preferences on banks
   - imposing “objective” (incomplete + inertia) preferences might lead to market breakdown
   - argument in favor of “subjective” (complete, pessimistic) preferences
5. Case Study: Knightian uncertainty remains uninsured
Case Study: Computations

Note that

$$-E \exp (-U_t + \alpha U_t - \alpha^2 / 2t) = -\exp ((1/2 - \alpha) t)$$

Hence, agent 1 prefers $U$ to 0 for $\alpha > 1/2$ and prefers full insurance to $U$ for $\alpha < 1/2$. So, full insurance is not better than keeping $U$. 