Market Breakdown and Indeterminacy under Model Uncertainty

Frank Riedel Rose–Anne Dana

[1]Center for Mathematical Economics
Bielefeld University

[2]CEREMADE
Université Paris–Dauphine

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Outline

Uncertainty versus Risk

- Roulette versus Horse Races
- objective probability versus no probabilities, just uncertain outcomes
- $(\Omega, \mathscr{F}, P)$ probability space versus $(S, \mathscr{S})$ measurable space, $X : (S, \mathscr{S}) \to \mathbb{R}$

Savage, Anscombe–Aumann

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- events $A, B \subset S$,
  $1_A \succ 1_B \Leftrightarrow$ "A" is more probable than "B"
  $\Leftrightarrow P(A) > P(B)$ for some *subjective* probability P

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Uncertainty II: Ellsberg

Asking for exact subjective probabilities too demanding in many real–world decisions.

Example

Imagine a baseball fan. He has the choice between two bets.
Situation 1:

- SF Giants win the World Series
- Arminia Bielefeld wird Pokalsieger 2013.

Situation 2:

- SF Giants do not win the World Series
- Arminia Bielefeld wird nicht Pokalsieger 2013.

It is perfectly rational to go for the first bet in both cases; but this would contradict the additivity of probability.

Uncertainty III: Examples

Knight (1921): many economic decisions are of a one–shot nature and one cannot presume probabilities

- Probability fairly well known for
  - Car Insurance
  - Life Insurance (Mortality Risk)
  - "IBM"
- probability less clear for
  - market entry
  - patents
  - Startups
  - Rating 'BB'

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Uncertainty IV: Two Formal Models

As "P" is not exactly known, work with a whole class of probability measures $\mathscr{P}$, (*Huber*, 1982, Robust Statistics)

Bewley's Model: uniform multiple prior approach

- incomplete expected utility
- $X \preceq Y \Leftrightarrow \forall P \in \mathscr{P}\, E^P u(X) \leq E^P u(Y)$
- plus inertia
- agents move from status quo ω^i to x^i iff $E^P u(x^i) > E^P u(\omega^i)$ for all priors $P \in \mathscr{P}^i$

Maximin Utility Model: Extreme Multiple Prior

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Von Neumann-Morgenstern Parameters: Multiple Priors

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Variational Utility Models: Pessimistic Multiple Priors

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Variational Utility Models: Pessimistic Multiple Priors

- complete preferences, pessimistic approach:
- $U(X) = \min_{P \in \mathscr{P}} E^P u(x)$

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Our paper

- Complete General Equilibrium Analysis of Dynamic Economies with Incomplete Expected Utility Preferences and Inertia
- Case Study: Market Breakdown and Inertia, Indeterminacy

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Literature

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Model: Bewley and Savage Economies

Definition

1. B–economy = Standard dynamic exchange economy under uncertainty, except for incomplete multiple–prior preferences given by a set of priors $\mathscr{P}^i$ for agent i
2. Fix priors $Q^i \in \mathscr{P}^i$.
   S–economy with priors $Q = (Q^1, Q^2, \ldots, Q^I)$= complete preferences, and possibly heterogeneous priors $Q = (Q^1, Q^2, \ldots, Q^I)$
3. S for Savage, not a risk economy!

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Model ctd.

- I agents with multiple priors $\mathscr{P}^i$
  - priors admit densities with respect to a reference measure P^0
  - agents agree on null sets
  - for $Q^i \in \mathscr{P}^i$, we denote the density process by q_t^i
- Consumption plans $c^i = (c_t^i(\omega))$ for $t = 0, 1, \ldots, T$
- agent i weakly prefers c^i over d^i iff

$$\text{for all priors } Q \in \mathscr{P}^i \quad E^Q \sum_{t=0}^{T} u^i\left(t, c_t^i\right) \geq E^Q \sum_{t=0}^{T} u^i\left(t, d_t^i\right)$$

- u^i nice period utility function
- endowments $\omega^i = (\omega_t^i(\omega))$ are strictly positive
- focus on interior allocations

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Efficiency

- An allocation c consists of consumption plans $c^i = \left(c_t^i(\omega)\right), i = 1, \ldots, I$
- c is feasible if $\sum_i c^i = \sum_i \omega^i$
- c is efficient if there is no other feasible allocation d such that $d^i \succ^i c^i$ for all agents i

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Efficiency in Savage Economies

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- Fix priors $Q = (Q^1, Q^2, \ldots, Q^I)$
- A feasible interior allocation $c = (c^1, c^2, \ldots, c^I)$ is efficient in the S–economy with priors $Q = (Q^1, Q^2, \ldots, Q^I)$ iff the marginal rates of substitution of all agents coincide, i.e.

$$MRS_t^i(Q^i) = \frac{u_c^i(t, c_t^i) q_t^i}{u_c^i(c_0^i)} = \frac{u_c^j(t, c_t^j) q_t^j}{u_c^i(c_0^j)} = MRS_t^j(Q^j)$$

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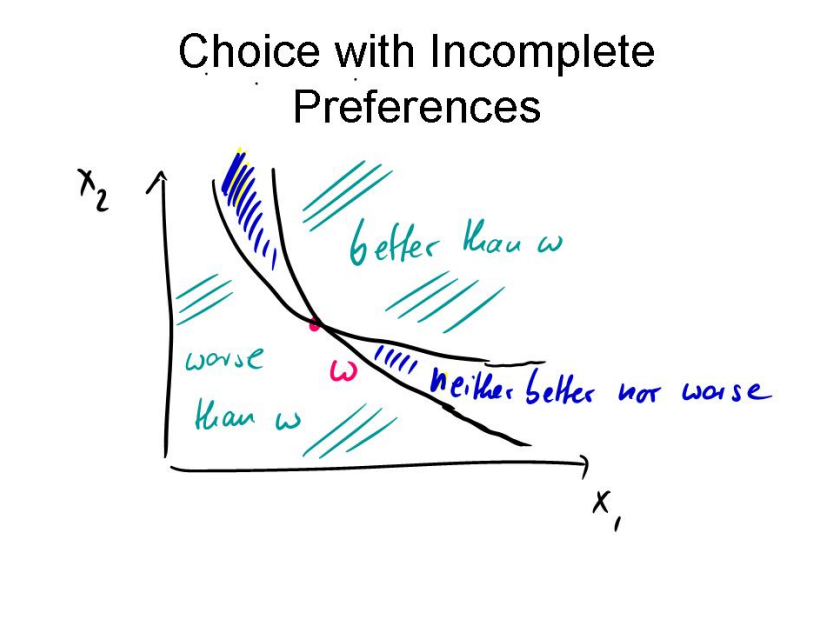
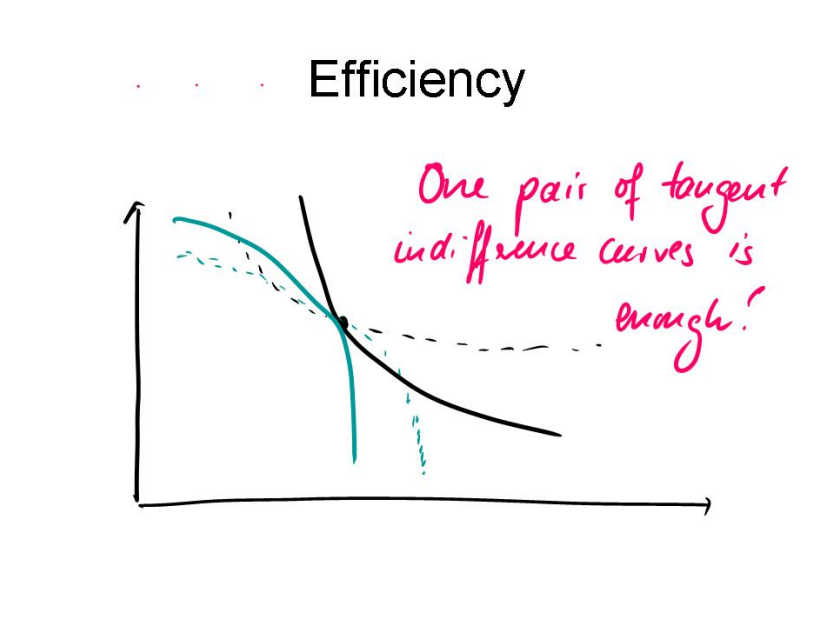
Efficiency in Bewley economies

More or less trivial:

Lemma

if c is efficient in some S–economy with priors Q, then c is efficient in the B–economy.





- Challenge: the converse!

Efficiency in Bewley economies

MRS=Risk–Adjusted Prior + Subjective Interest Rate

- Every MRS can be written as

$$MRS_t^i = \frac{u_c^i(t, c_t^i) q_t^i}{u_c^i(c_0^i)} = M_t^i \exp\left(-\sum_{s=1}^{t} r_s^i\right)$$

  for a martingale $M^i = M^i(Q^i)$ with expectation 1 and a subjective interest rate r^i
- Interest rate is predictable
- Decomposition is unique (Multiplicative Doob Decomposition)
- M^i density process of a new measure, the risk–adjusted prior or equivalent martingale measure

Efficiency in Bewley economies

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$$MRS_t^i = \frac{u_c^i(t, c_t^i) q_t^i}{u_c^i(c_0^i)} = M_t^i \exp\left(-\sum_{s=1}^{t} r_s^i\right)$$

  for a martingale $M^i = M^i(Q^i)$ with expectation 1 and a subjective interest rate r^i
- Interest rate is predictable
- Decomposition is unique (Multiplicative Doob Decomposition)
- M^i density process of a new measure, the risk–adjusted prior or equivalent martingale measure

Efficiency in Bewley economies

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For agent i, let

$$\mathscr{MRS}^i = \left\{ MRS^i(Q^i) : Q^i \in \mathscr{P}^i \right\}$$

be the set of agent i's marginal rates of substitution.
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be the set of agent i's equivalent martingale measures.

Efficiency in Bewley economies

Theorem

An interior allocation c is efficient in the Bewley economy if and only if one of the following conditions holds true:

1. *the agents' share a common marginal rate of substitution,* $\bigcap_i \mathscr{MRS}^i \neq \emptyset$
2. *the agents share a risk–adjusted prior and for a common risk–adjusted prior Q all individual interest rates are equal, i.e.*

$$r^i(Q, c^i)_t = r^j(Q, c^j)_t$$

   for all $i, j = 1, \ldots, I$ and $t = 0, \ldots, T$,
3. *for some selection of priors $Q^i \in \mathscr{P}^i$, $i = 1, \ldots, I$, c is efficient in the Savage economy with priors $Q = (Q^1, \ldots, Q^I)$.*

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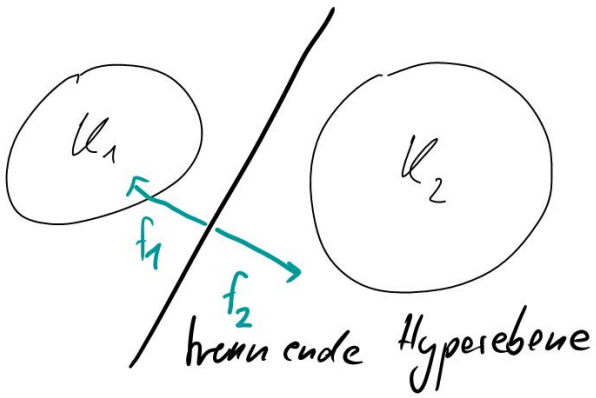
Samet's Theorem

Theorem (*Samet, Games and Economic Behavior*, 1998)

Let $K_1, \ldots, K_n$ be convex, closed, nonempty subsets of the simplex Δ^m in $\mathbb{R}^m$.
$\bigcap K_i = \emptyset$ iff there are $f_1, \ldots, f_n \in \mathbb{R}^m$ such that $\sum f_i = 0$, and $f_i \cdot x_i > 0$ for each $x_i \in K_i, i = 1, \ldots, n$.

Separation Theorem



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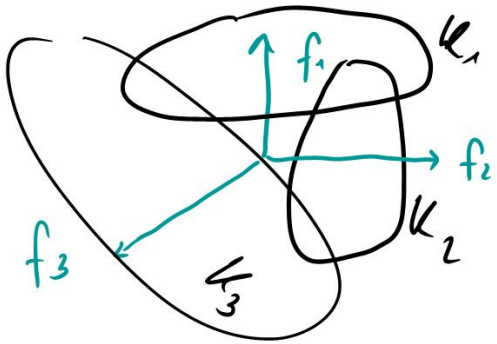

Samet's Theorem for L^∞

Theorem

Let $(S, \mathscr{S}, P)$ be a probability space. Let $(K_i)_{i=1,\ldots,n}$ be nonempty, convex, and $\sigma(L^1(S, \mathscr{S}, P), L^\infty(S, \mathscr{S}, P))$-compact subsets of $\Delta = \left\{D \in L^1_+(S, \mathscr{S}, P) : E\,D = 1\right\}$.
Then $\bigcap K_i = \emptyset$ if and only if there exists $g_i \in L^\infty(S, \mathscr{S}, P)$ with $\sum g_i = 0$ such that $\int g_i x_i dP > 0$ for all $x_i \in K_i, i = 1, \ldots, n$.

Equilibria in Bewley economies

An (Arrow–Debreu) equilibrium consists of a (state) price process $p = (p_t)$ (adapted), and a feasible allocation c such that for each agent i

$$d^i \succ^i c^i \Rightarrow E^{P_0} \sum_t p_t d_t^i > E^{P_0} \sum_t p_t c_t^i$$

Corollary

Any interior equilibrium (p^*, c^*) of the Bewley economy is an interior equilibrium for some Savage economy with priors $Q^i \in \mathscr{P}^i, i = 1, \ldots, I$ and vice versa.

Remark

- *Huge number of equilibria if uncertainty is nontrivial*
- *Indeterminacy (compare Rigotti–Shannon)*

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Equilibria with Inertia: Existence

- Inertia: agents choose $c^i \neq \omega^i$ only if they strictly prefer c^i over ω^i under all $P \in \mathscr{P}^i$
- Big reduction of number of equilibria
- New Idea: introduce a certain class of variational preferences (*Maccheroni, Marinacci, Rustichini*) with reference level ω^i

$$V^i(x) = \min_{Q \in \mathscr{P}^i} E^Q\left((U^i(x) - U^i(\omega^i))\right) \tag{1}$$

Theorem (Existence)

Any equilibrium of an economy with complete variational preferences (1) *is an equilibrium with inertia (in the B–economy). In particular, equilibria with inertia exist.*

Continuity Lemma

Such variational preferences are Mackey–continuous.

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No Trade and Indeterminacy

CAPM–like Model with Uncertainty

- No aggregate uncertainty
- individual endowments depend on risky source (distribution known) and uncertain source (distribution unknown)
- agents agree that risk and uncertainty are independent
- Constant absolute risk aversion and normal distributions incomplete mean–variance model

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Results in a Nutshell

- risk is completely insured, and thus there is always trade,
- uncertainty is fully insured if ambiguity is small,
- when ambiguity exceeds a certain critical level, the full insurance allocation (where also the uncertain part of the endowment is fully insured) is *not* an equilibrium with inertia,
- and there is an equilibrium with inertia where subjective uncertainty is not insured at all
- inertia and uncertainty lead to no trade
- finally, although inertia is a strong equilibrium refinement, we nevertheless have indeterminacy of equilibria with inertia

Case Study: Details

- two agents with CARA utility, $u^i(x) = -exp(-x)$
- aggregate endowment is zero
- agent 1 has endowment $\omega_t^1 = R_t + U_t$
- R is risky and U is uncertain
- $R_t = \sum_{s=1}^t \varepsilon_s, \quad \varepsilon_s \sim N(0,1)$, i.i.d.
- $U_t = \sum_{s=1}^t \nu_s$, (ν_t) independent experiments with identical ambiguity
- time–consistent dynamic model of multiple priors
- $$q_t = \exp\left(\sum_{s=1}^t \left(\alpha_s \nu_s - \frac{1}{2}\alpha_s^2\right)\right)$$
  for some U–predictable process (α_s) with values in $[-\kappa, \kappa]$
- why only U–predictable? Agents agree on independence of R and U under all priors!

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equilibrium with expected utility and pessimistic multiple priors

- Textbook knowledge: with homogeneous priors and expected utility, full insurance in equilibrium
- both risk and uncertainty are fully insured
- *Billot, Chateauneuf, Gilboa, Tallon*: also with Gilboa–Schmeidler preferences if agents share at least one prior
- (argument carries over to dynamic settings)
- intuition: for a pessimist, full insurance is better than uncertainty

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Case Study: Full Insurance is No Equilibrium with Inertia under High Uncertainty

- Full Insurance of *both* risk and uncertainty is an equilibrium in the S–economy with homogeneous priors
- hence an equilibrium in the B–economy
- but not an equilibrium with inertia if uncertainty is high
- Intuition: every agent finds one very optimistic prior where she prefers uncertainty over insurance
- ... positive mean
- ... outweighs keeping risk
- $\kappa > 1$

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Equilibria

An equilibrium consists of an adapted, bounded price process (p_t) and a feasible allocation $c = (c_t^i)$ such that the budget constraint

$$E^{P_0} \sum_t p_t (c_t^i - \omega_t^i) = 0$$

is satisfied.
(p, c) is an equilibrium with inertia if for each agent i we have either $c^i = \omega^i$ or

$$E^Q \sum_t u^i(t, c_t^i) > E^Q \sum_t u^i(t, \omega_t^i)$$

for all $Q \in \mathscr{P}^i$

Equilibria

Lemma

Let $(\alpha_s), (\beta_s)$ *be two* $\mathscr{F}^U$*–predictable processes with values in* $[-\kappa, \kappa]$*. The allocations* $(c^1, -c^1)$ *with*

$$c_t^1 - c_0^1 = \frac{1}{2}\left(\sum_{s=1}^{t}(\alpha_s - \beta_s)\,\nu_s + \frac{\beta_s^2 - \alpha_s^2}{2}\right) \tag{2}$$

are efficient in the B–economy. For properly chosen c_0^1 *and price* p*, they form an equilibrium.*
In particular for $|\alpha| \leq \kappa$*, the allocation* $(\alpha U_t, -\alpha U_t)$ *with the deterministic price* $p_t = \exp\left(-(\rho + 1/2\alpha^2)t\right)$ *is an equilibrium.*

Remark

indeterminacy of the interest rate in equilibrium: $\rho + \frac{\alpha^2}{2}$ *for* $\alpha \in [-\kappa, \kappa]$*.*

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Equilibria, ctd.

Theorem

In every equilibrium (p, c) of the B–economy, risk is fully insured in the sense that the equilibrium allocation c depends only on U not on R.

Equilibria with Inertia, Full Insurance

Theorem

For sufficiently small levels of ambiguity, namely $\kappa < 1$, the full insurance allocation $c^1 = c^2 = 0$ is an equilibrium allocation with inertia.
If ambiguity is sufficiently large, $\kappa \geq 1$, the full insurance allocation $c^1 = c^2 = 0$ is not an equilibrium allocation with inertia.

Equilibria with Inertia, No Insurance of Uncertainty

Theorem

For large levels of uncertainty, $\kappa \geq 1$, there is an equilibrium with inertia in which agent 1 consumes

$$c_t^1 = U_t \, .$$

The equilibrium price is

$$p_t^* = \exp\left(-\left(\rho + \frac{1}{2}\right) t\right) \, .$$

Case Study: Equilibrium, Uniqueness

Theorem

The equilibria with inertia are indeterminate.

For $\kappa < 1$, there exists a continuum of equilibria with inertia with allocation $(c^1, -c^1)$ and $c^1 = \varepsilon U$ for sufficiently small $\varepsilon \neq 0$.
For $\kappa \geq 1$, there exists a continuum of equilibria with inertia with allocation $(c^1, -c^1)$ and $c^1 = (1 - \varepsilon)U$ for sufficiently small $\varepsilon \neq 0$.

Intuition:
The insurance of R gives a positive utility gain. This allows to trade some parts without violating the inertia constraint.

A Related Motivation: Control of Investment Banks

Regulation of Financial Markets

Regulation can be interpreted as "imposing preferences"

- stress testing: accept a deal only if it performs better than status quo in all tests $\Leftrightarrow$ Bewley with inertia
- worst–case approach : compare the worst–case outcomes deal versus status quo and accept a deal if the worst–case outcome of the deal is better the the worst–case outcome of the status–quo
  - Coherent Risk Measures (*Artzner, Delbaen, Eber, Heath*) $\Rightarrow$ Gilboa–Schmeidler
  - Convex Risk Measures (*Föllmer, Schied, Frittelli, Rosazza Gianin*) $\Rightarrow$ Variational Preferences

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Conclusion

1. General Equilibrium Analysis for Bewley's Incomplete Preference Approach
2. Link to Variational Expectations
3. Samet's Theorem for L^∞
4. Link to Regulation of Financial Markets:
   - Regulation is a way to impose preferences on banks
   - imposing "objective" (incomplete + inertia) preferences might lead to market breakdown
   - argument in favor of "subjective" (complete, pessimistic) preferences
5. Case Study: Knightian uncertainty remains uninsured

Case Study: Computations

Note that

$$-E\exp\left(-U_t + \alpha U_t - \alpha^2/2t\right) = -\exp\left(\left(1/2 - \alpha\right)t\right)$$

Hence, agent 1 prefers U to 0 for $\alpha > 1/2$ and prefers full insurance to U for $\alpha < 1/2$. So, full insurance is not better than keeping U.