Asset Demand Risk and Demand Discovery

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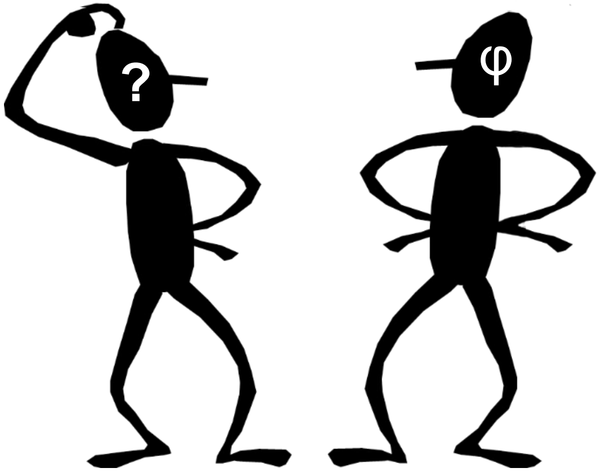


Introduction

- Investors trade dynamically over time
  - Smooth consumption and share risks
  - But exposed to future pricing risk
- Sources of future pricing risk
  - If future asset demand functions are fixed and common knowledge, then future price function P is known. Only future cash-flow states not known. No asset demand risk.
  - If future asset demand functions vary over time due to a common knowledge sentiment factor, then future prices are a function P of future cash-flow and sentiment factors. Asset demand risk but no sentiment inference.
  - If future asset demand functions vary over time and sentiment is ex ante private information, then asset demand risk + sentiment inference
- New features
  - Asset demand risk
  - Demand discovery

Asset demand risk and demand discovery are likely

- Retail investor asset demand
  - Utility functions depend on genetics and life experiences
  - Private budget constraints
- Institutional investors and traders
  - Internal incentive structure
  - Internal funding, capital adequacy, and risk-limit constraints
- Utility functions and investment constraints are high dimensional
  - $U: R \rightarrow R$ i.e., maps consumption level → utility.
  - Utility functions live in a big space of continuous, increasing, concave functions.
  - Can change over time and can depend
- Investors are likely to have better info about self than others
  - Does need to be perfect self-knowledge, just some is enough.

Questions

- How much preference info is resolved by demand discovery?
  - Full revelation? Pooling?
- How general are equilibrium pooling outcomes?
  - Knife-edge? Strong assumptions?
- Does asset demand risk affect pricing?
  - Are pooling equilibrium prices and trades different from pooling prices and trades?
- Impact on risk premia and asset price volatility?
  - How does asset demand risk affect return volatility?
  - Is there an asset demand risk premium?
  - How does asset demand risk affect cash flow risk premium? Reverse effect?

This paper

- General results
  - Asset demand uncertainty only possible if market is statically cash-flow incomplete
  - Challenge: Even just proving existence of equilibrium can be hard
  - Two proof strategies
    1. Identify general conditions under which, if equilibrium exists, cannot beFR. Hence, equilibrium must involve asset demand risk.
    2. Posit market with well-behaved equilibrium given CK investor preferences. Identify conditions s.t. equilibrium exists with asset demand risk once private preference knowledge.
  - Some results
    1. FR equilibrium requires set of ex ante possible types Φ to be sufficiently constrained.
    2. If set Φ includes a convex subset, then, if equilibrium exists, cannot be FR.
    3. Asset demand risk matters generically for asset pricing if the preference uncertainty is not fully revealed by demand discovery.
    4. Analytically tractable example model

This paper (2)

- Numerical results (in progress)
  - Single stock, 3 dates
    - Substantial price volatility from asset demand risk
    - Large shadow risk premium.
  - Stock + bill
    - Currently being completed.

Literature

- Canonical asset pricing
  - Fixed CK investor preferences
    - Lucas [1978], Merton [1973], Duffie and Huang [1985]
- Sentiment-based asset pricing
  - Stochastic but CK investor preferences
    - De Long, Shleifer, Summers and Waldman [1990], Lettau and Wachter [2011]
- Asymmetric information about future cash flows
  - CK investor preferences except for simple noise traders
    - Grossman and Stiglitz [1981]], Wang [1993], Detemple [2002]
- Demand discovery
  - Investor preferences are private knowledge and change over time
    - Grossman [1988], Kraus and Smith [1989], GHS [2005], Grundy and McNichols [1989]

Model

- Dates 0, 1, …, T
  - Asset pricing not transactional time scale

- Traded securities
  - N-1 long-lived traded securities with unit outstanding suppy
  - Dividends $d_{1,wt}, \ldots, d_{N-1,wt}$,
  - Discrete-time, discrete-state cash-flow state tree
  - Generic cash-flow state ω_t. Specific state $\omega_{t,j}$.
  - Controls current dividends at date t and also future cash-flow subtree.
  - Probability $g(\omega_t)$

  - 1-period zero-net supply risk-free bill paying $d_{N,wt} = 1$ at each date t
  - P_{wt} is vector of N traded-security prices

Investors

- Informed investors
  - Unit mass, price-takers
  - Lifetime utility

$$v(c_0^I) + \sum_{t=1,\ldots,T} \beta^t \, \mathrm{E}_0^I[\varphi(t,\omega_t)v(c_t^I)]$$

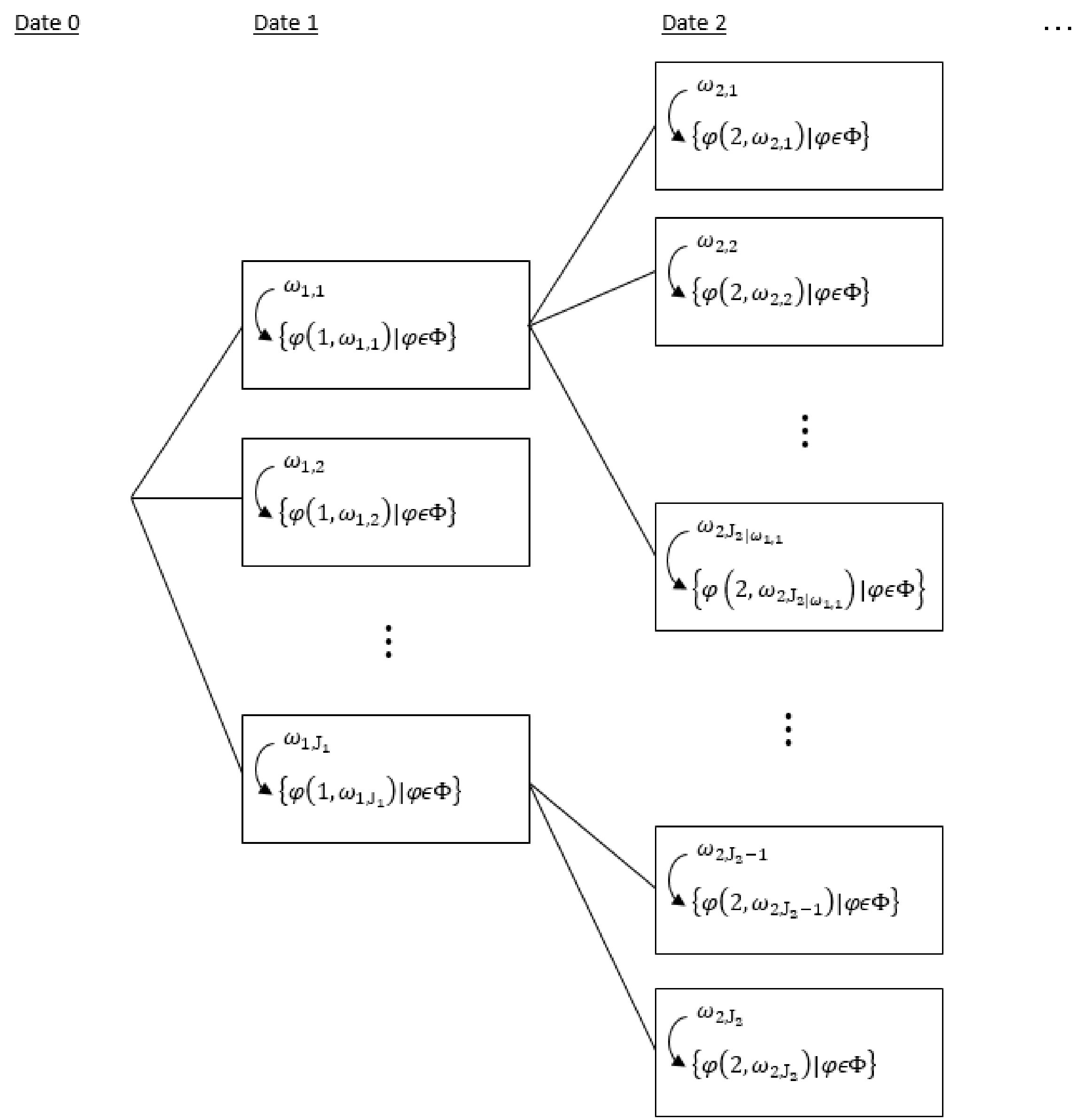
  - State contingent preference factor $\varphi(t, w_t)$ in state w_t. Profile $\varphi = \{\varphi(t, w_t)\}$
  - Greed/fear? Patience/impatience? Macro wealth effects?

- Uninformed investors
  - Unit mass, price-takers
  - Lifetime utility

$$u(c_0^U) + \sum_{t=1,\ldots,T} \beta^t \, \mathrm{E}_0^U[u(c_t^U)]$$

  - Don't know φ. Do know $\varphi \in \Phi$ where probability belief is $f(\varphi) > 0$

Cash-flow tree & preference uncertainty



More model

- Traded-security holdings
  - Informed investor: θ_{wt}^{I}
  - Uninformed investor: θ_{wt}^{U}
  - Market clearing
- Non-tradable consumption-good endowments
  - Informed investor: e_{wt}^{I}
  - Uninformed investor: e_{wt}^{U}
- Traded-security price function
  - $P(t, w_t, \varphi, \theta_{wt})$
  - Cash flow risk: Both investors uncertain about future cash-flow state w_t.
  - Asset demand risk:
    - <u>Uninformed</u> investors <u>don't</u> know type φ and, thus, do not know P function.
    - <u>Informed</u> investors <u>do</u> know φ, and do know P function.

Beliefs

- Updated cash-flow probabilities
  - $g_{wt}(w_s)$
  - Bayes Rule given exogenous cash-flow state dynamics
  - Common knowledge

- Updated preference types probabilities
  - $f_{wt}(\varphi)$
  - Bayes Rule given endogenous information revealed by informed investors via the trading process
  - Only uninformed investors learn through demand discovery

Definition

Rational expectations equilibrium (REE) is collection of processes (p, θ) such that

- traded-security price process p clears consumption-good and asset markets and
- the asset-holding processes θ^I and θ^U maximize lifetime expected utility for informed and uninformed investors subject to budget constraints and given rational beliefs about prices given investors' respective information.

Informed investor's problem

- FOCs

$$P_0 = \sum_{s=1,\ldots,T} \sum_{j=1,\ldots,J_s} \beta^t \, \varphi(s, \omega_{s,j}) \, \frac{v'(c^I(s, \omega_{s,j}, \varphi, \theta_{s-1}))}{v'(c^I(0, \omega_0, \varphi, \theta_{-1}))} \, g(\omega_{s,j}) \, d_{\omega_{s,j}}$$

- Implicit state prices

$$\pi_0(\omega_{s,j}) = \beta^t \, \varphi(s, \omega_{s,j}) \, \frac{v'(c^I(s, \omega_{s,j}, \varphi, \theta_{s-1}))}{v'(c^I(0, \omega_0, \varphi, \theta_{-1}))} \, g(\omega_{s,j})$$

- State price valuation representation

$$P_0 = \sum_{s=1,\ldots,T} \sum_{j=1,\ldots,J_s} \pi_0(\omega_{s,j}) \, d_{\omega_{s,j}}$$

Informed investor's problem (2)

- Price process p = {P_{wt}} over time

$$P_{\omega_t} = \sum_{s=t+1,\ldots,T} \; \sum_{j=1,\ldots,J_{s|\omega_t}} \pi_{\omega_t}(\omega_{s,j})\, d_{\omega_{s,j}}$$

- Conditional state prices $\pi_{wt}(w_s) = \pi_0(w_s) / \pi_0(w_t)$

1st preference learning channel

- State price linear algebra channel
  - N traded-security prices at any date/state give N equations in J unknown state prices
  - Let $\Pi(P_0)$ = set of possible state prices π given prices P_0
  - Each of these possible state prices implies a traded-security price process *p*
  - Let $\mathcal{P}(P_0)$ = set of possible traded-security price processes given P_0

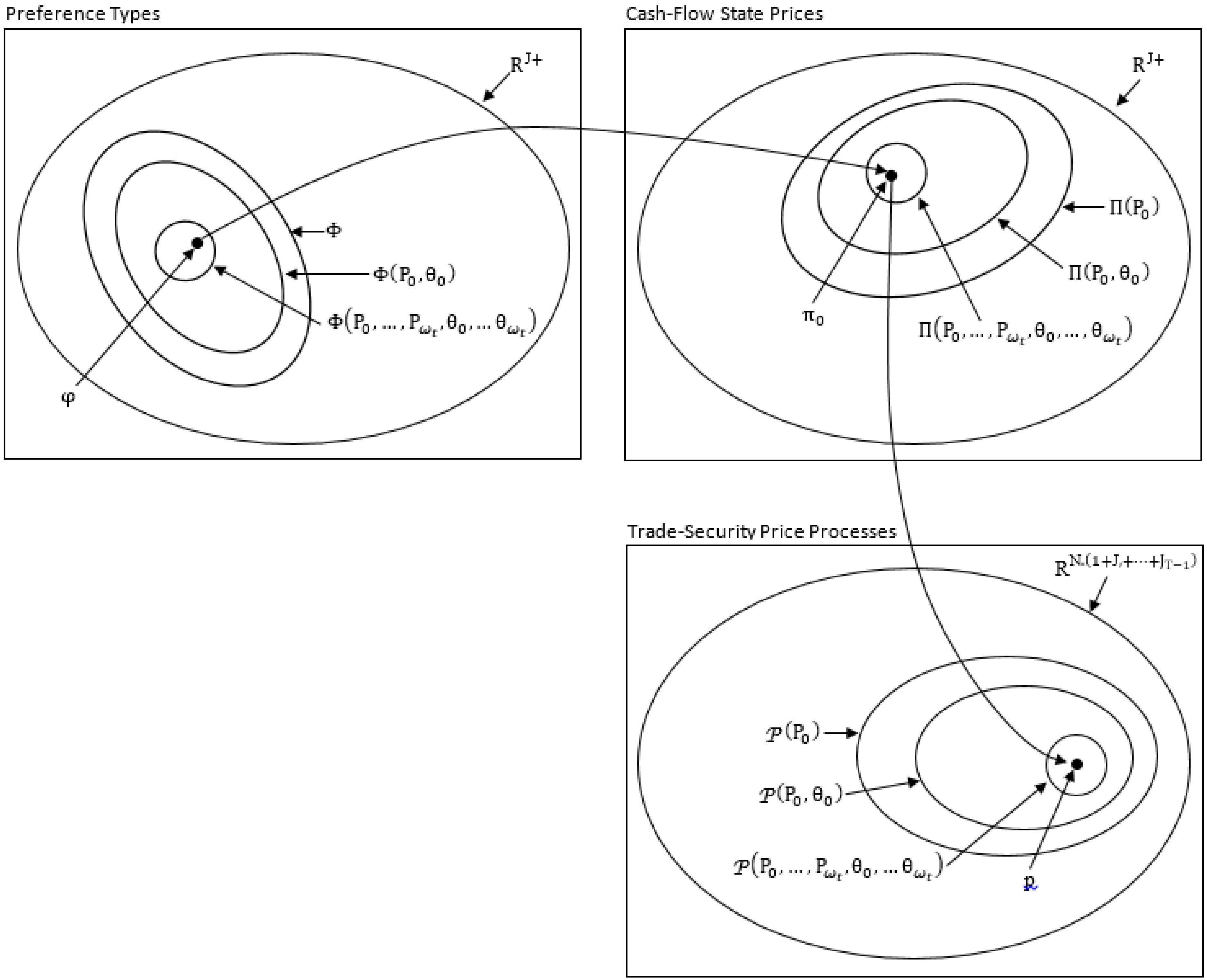
- Proposition 1

  - If the future traded-security cash flows after each state w_t are linearly independent for all dates t = 0, . . . T −1 and if $J_{t+1|wt} \geq 2$ (i.e., there are at least two subsequent sub-trees) for all w_t, then simply observing the traded-security price history over time is insufficient, without knowledge of Φ, to infer the equilibrium state prices π_0 exactly at any date $t < T - 1$.

2^{nd} preference learning channel

- Equilibrium beliefs channel
  - Uninformed investors know the implied state prices π must be consistent with informed investor's FOCs.
  - For each type $\varphi \in \Phi$, exists an equilibrium consumption and traded security price process
  - Let $\Phi(P_0, \theta_0)$ = set of $\varphi \in \Phi$ such that there is a possible informed investor would hold observed θ_0 at observed prices P_0.
  - Let $\Pi(P_0, \theta_0)$ = set of π given FOCs for types $\varphi \in \Phi(P_0, \theta_0)$
  - Let $\mathcal{P}(P_0, \theta_0)$ = set of possible traded-security price processes

Preferences φ, state prices π, and price processes *p*



Uninformed investor problem

- FOCs at date 0

$$P_0 = \sum_{s=1,\ldots,T} \sum_{j=1,\ldots,J_s} \left[\int_{\varphi \in \Phi(P_0,\theta_0)} \beta^s \frac{u'(c^U(s,\omega_{s,j},\varphi,\theta_{s-1}))}{u'(c^U(0,\omega_0,\varphi,\theta_{-1}))} f_0(\varphi)\, d\varphi \right] g(\omega_{s,j})\, d_{\omega_{s,j}}$$

$$f_0(\varphi) = \frac{f(\varphi)}{\int_{\varphi \in \Phi(P_0,\theta_0)} f(\varphi)\, d\varphi}$$

- FOCs at later dates/states

$$P_{\omega_t} = \sum_{s=t+1,\ldots,T} \sum_{j=1,\ldots,suc(s,\omega_t)} \left[\int_{\varphi \in \Phi(P_0,\ldots,P_{\omega_t},\theta_0,\ldots,\theta_{\omega_t})} \beta^{s-t} \frac{u'(c^U(s,\omega_{s,j},\varphi,\theta_{s-1}))}{u'(c^U(t,\omega_t,\varphi,\theta_{t-1}))} f_{\omega_t}(\varphi)\, d\varphi \right] g(\omega_{s,j})\, d_{\omega_{s,j}}$$

$$f_{\omega_t}(\varphi) = \frac{f(\varphi)}{\int_{\varphi \in \Phi(P_0,\ldots,P_{\omega_t},\theta_0,\ldots,\theta_{\omega_t})} f(\varphi)\, d\varphi}$$

Market incompleteness

- Definition
  - A market is *static cash-flow complete* if, for each future cash-flow state w_t at each date t, there is a buy-and-hold trading strategy at date 0 using traded securities that replicates an Arrow-Debreu security paying $1 in cash-flow state w_t.
- Proposition 2
  - If a market is statically cash-flow complete, then there is no asset demand risk.

FR equilibria and restrictions on Φ

- Proposition 3
  - A fully-revealing equilibrium does not exist unless the set Φ of possible informed-investor types φ is sufficiently restricted a priori.
  - Intuition: Can always construct a type $\widehat{\varphi}$ who would pool with a type φ^*.

$$\hat{\varphi}(t, \omega_{t,j}) = \frac{\hat{\pi}_0(\omega_{t,j})}{g(\omega_{t,j})} \cdot \frac{v'(c_0^{I,\hat{p}})}{v'(c_{t,j}^{I,\hat{p}})}$$

  - Requires common knowledge about how uninformed investors will act if they are surprise by a trading outcome in the future

FR equilibria and restrictions on Φ (2)

- Proposition 4
  - If i) the set Φ of ex ante possible preferences includes a non-degenerate convex subset and ii) if the traded-security cash flows are linearly independent going forward from each date t and state w_t, then, if an equilibrium exists in which iii) the uninformed investors' asset demands are continuous in arbitrage-free prices, then it is not a FR equilibrium given trading at date 0.
  - Intuition: Again, can always construct a type $\widehat{\varphi}$ in convex subseq who would pool with a type φ^*.

Demand Uncertainty Irrelevance

- Definition
  - A pooling equilibrium exhibits demand uncertainty irrelevance (DUI) if, for each preference φ in Φ(P0, θ0), the CK equilibrium corresponding to φ clears at the same date-0 prices and trades, P_0 and θ_0, as in the pooling equilibrium.

- Proposition 6
  - Consider a pooling equilibrium with a finite number $K(P_0, \theta_0)$ of types φ in the pool $\Phi(P_0, \theta_0)$ at date 0. Suppose also that this equilibrium becomes fully revealing at date 1. Asset demand risk matters generically for date-0 pricing in that the set U^{DUI} of uninformed preferences leading to DUI-pooling equilibria with N traded securities is a lower-dimensional subset of the set U^{pool} of uninformed preferences that lead to pooling equilibria.

Uninformed investor

- Date-0 FOCs in pooling equilibrium

$$P_0 = \sum_{t=1,\ldots,T} \sum_{j=1,\ldots,J_t} \sum_{\varphi \in \Phi_0} \beta^t m(c_t^U(\varphi, \omega_{t,j}), c_0^U)\, f(\varphi)\, g(\omega_{t,j})\, d_{\omega_{t,j}}$$

  - The m's are MRS for the uninformed investor.
  - N equations in J unknowns at date 0.

Uninformed investor

- Date-0 FOCs in CK equilibrium

$$P_0^{CK}\, u'(c_0^{U,CK}) = \sum_{t=1,\ldots,T}\ \sum_{j=1,\ldots,J_t} \beta^t u'(c_t^{U,CK}(\varphi,\omega_{t,j}))\, g(\omega_{t,j})\, d_{\omega_{t,j}}$$

- If DUI

$$P_0 = \sum_{t=1,\ldots,T}\ \sum_{j=1,\ldots,J_t} g(\omega_{t,j})\, \beta^t m(c_t^{U,CK}(\varphi,\omega_{t,j}), c_0^U)\, d_{\omega_{t,j}}$$

- But since pool is fully revealing at date 1

$$m(c_t^{U,CK}(\varphi,\omega_{t,j}), c_0^U) = m(c_t^U(\varphi,\omega_{t,j}), c_0^U)$$

  - N* K(P_0, θ_0) equations in J unknowns.
  - Thus, DUI uninformed-investor preferences are in a lower-dimensional linear subspace of the pooling uninformed-investor preferences

Example

- Assumptions
  - Single stock, no risk-free bill
  - Log preferences
  - Restrictions on consumption endowments
  - Three dates 0, 1, and 2

- Model can be solved explicitly in closed-form

FOCs

- Using FOCs + market-clearing at date 1 gives

$$P_1 = \frac{\beta\,(e_1^U + d_1\,(1-\theta_0^I)) + \mathrm{E}_1^I[\xi_2]\,(\beta(e_1^U + e_1^I + d_1) + e_1^I + d_1\theta_0^I)}{1 + \beta\,\theta_0^I + \mathrm{E}_1^I[\xi_2]\,(1-\theta_0^I)}$$

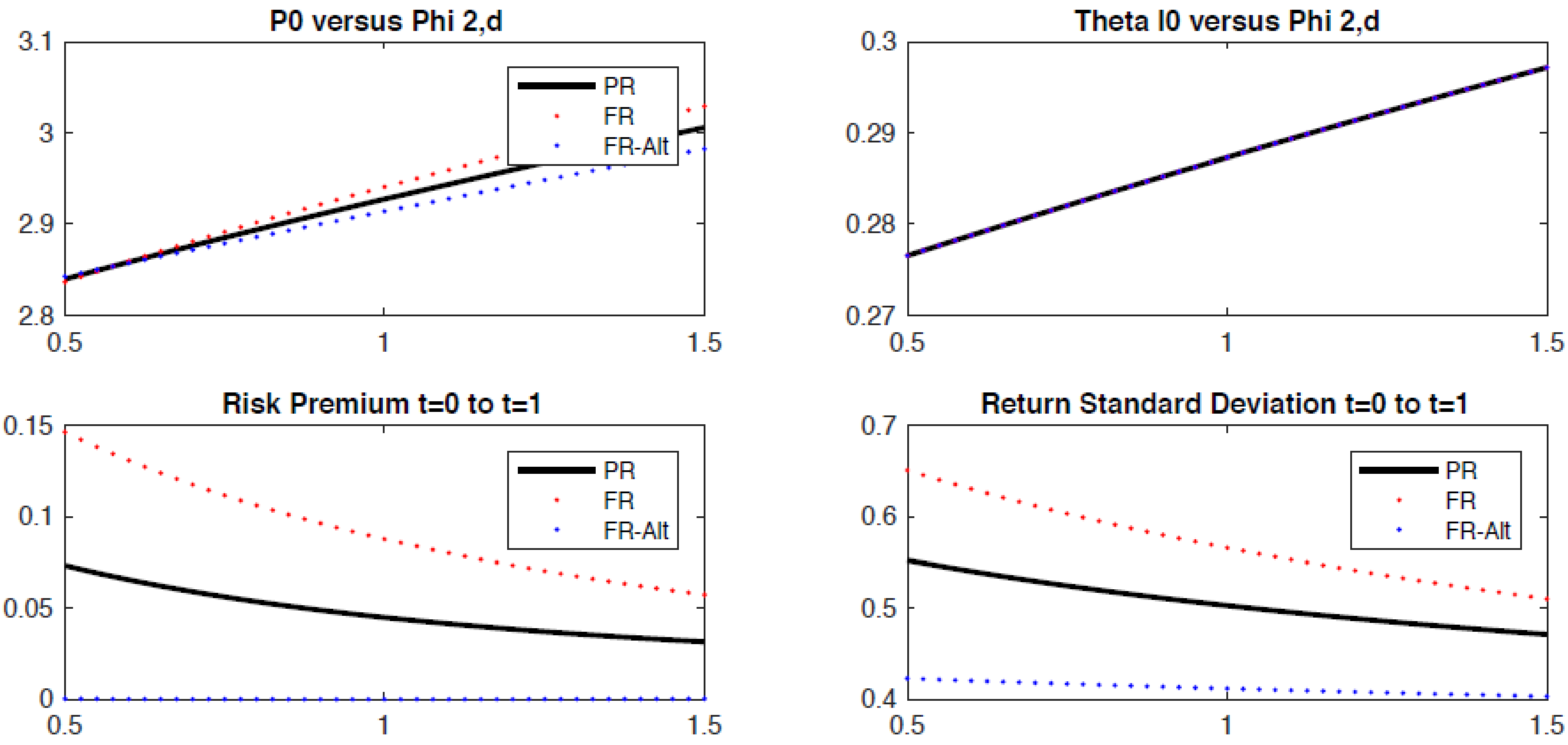
$$\theta_1^I = \frac{\mathrm{E}_1^I[\xi_2]\,([(1+\beta)d_1 + \beta e_1^U]\,\theta_0^I + e_1^I\,(1+\beta\theta_0^I))}{\beta\,[e_1^U + d_1\,(1-\theta_0^I)] + \mathrm{E}_1^I[\xi_2]\,(\beta[e_1^U + e_1^I + d_1] + e_1^I + d_1\theta_0^I)}$$

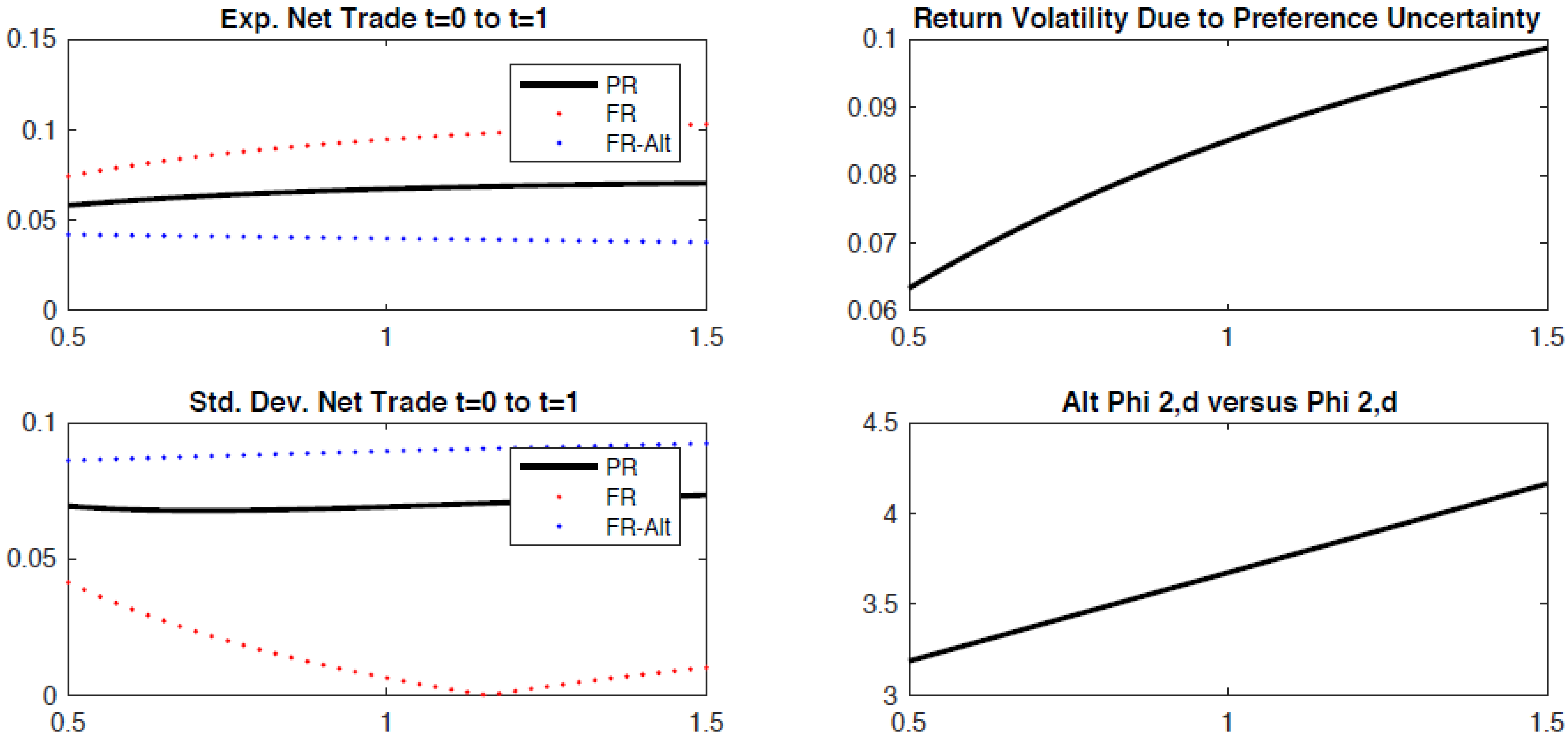
- FOCs with market clearing at date 0

$$\frac{P_0}{e_0^I + P_0(\theta_{-1}^I - \theta_0^I)} = E_0^I\left[\varphi_1 \frac{(1+\beta)d_1 + \beta e_1^U + [(1+\beta)(d_1 + e_1^I) + \beta e_1^U]\,E_1^I[\xi_2]}{e_1^I + [(1+\beta)d_1 + \beta(e_1^I + e_1^U)]\,\theta_0^I}\right]$$

$$\frac{P_0}{e_0^U + P_0(\theta_0^I - \theta_{-1}^I)} = \beta\,E_0^U\left[\frac{[(1+\beta)d_1 + \beta e_1^U]\,(1 + E_1^I[\xi_2]) + (1+\beta)\,e_1^I\,E_1^I[\xi_2]}{d_1(1-\theta_0^I) + e_1^U + (1-\theta_0^I)(d_1 + e_1^I + e_1^U)\,E_1^I[\xi_2]}\right]$$

Numerical results





Exp. Net Trade t=0 to t=1
PR
FR
FR-Alt
0.15
0.1
0.05
0
0.5
1
1.5
Return Volatility Due to Preference Uncertainty
0.1
0.09
0.08
0.07
0.06
0.5
1
1.5
Std. Dev. Net Trade t=0 to t=1
PR
FR
FR-Alt
0.1
0.05
0
0.5
1
1.5
Alt Phi 2,d versus Phi 2,d
4.5
4
3.5
3
0.5
1
1.5

Conclusions

- Asset demand risk and demand discovery seem like plausible and generic features of dynamic financial markets

- Generically, asset demand risk should matter for pricing and should be priced with a risk premium

- Lots of interesting extensions
  - Currently working on numerical models with non-log preferences and multiple traded securities
  - Make cash-flow state space continuous too
  - Symmetric investor type uncertainty & more than 2 groups

$$
\begin{aligned}
g_0^A\, v'(c_0^I(g_0^A,\theta_0)) &= \beta\,\varphi_1^A \sum_{j=1,\ldots J_1} v'(c_1^I(\theta_0,\omega_{1,j},\xi^A))\, g(\omega_{1,j})\, [d_{\omega_{1,j}} + P_1(\theta_0,\omega_{1,j},\xi^A)] \\
g_0^B\, v'(c_0^I(g_0^B,\theta_0)) &= \beta\,\varphi_1^B \sum_{j=1,\ldots J_1} v'(c_1^I(\theta_0,\omega_{1,j},\xi^B))\, g(\omega_{1,j})\, [d_{\omega_{1,j}} + P_1(\theta_0,\omega_{1,j},\xi^B)]
\end{aligned}
$$

$$
\varphi_1^B(\theta_0,\varphi_1^A) = \varphi_1^A\, h(\theta_0)
$$

$$
h(\theta_0) = \frac{\sum_{j=1,\ldots J_1} g(\omega_{1,j})\, v'(c_1^I(\theta_0,\omega_t,\xi^A))\, [d_{\omega_{1,j}} + P_1(\theta_0,\omega_t,\xi^A)]}{\sum_{j=1,\ldots J_1} g(\omega_{1,j})\, v'(c_1^I(\theta_0,\omega_t,\xi^B))\, [d_{\omega_{1,j}} + P_1(\theta_0,\omega_t,\xi^B)]}
$$

$$g_0^{U,pool}\, u'(c_0^U(g_0^{U,pool}, 1-\theta_0)) = \frac{f_a f(\varphi_1^A)\, G^A(\theta_0) + (1-f_a) f(\varphi_1^B(\theta_0, \varphi_1^A))\, G^B(\theta_0)}{f_a f(\varphi_1^A) + (1-f_a) f(\varphi_1^B(\theta_0, \varphi_1^A))}$$

$$G^A(\theta_0) = \sum_{j=1,\ldots,J_1} \beta u'(c_1^U(\theta_0, \omega_t, \xi^A))\, g(\omega_{1,j})\, [d_{\omega_{1,j}} + P_1(\theta_0, \omega_{1,j}, \xi^A)]$$

$$G^B(\theta_0) = \sum_{j=1,\ldots,J_1} \beta u'(c_1^U(\theta_0, \omega_t, \xi^B))\, g(\omega_{1,j})\, [d_{\omega_{1,j}} + P_1(\theta_0, \omega_{1,j}, \xi^B)]$$

Binomial/continuous equilibrium outcome