Activism, Strategic Trading, and Liquidity

Kerry Back
Pierre Collin-Dufresne
Vyacheslav Fos
Tao Li
Alexander Ljungqvist
Overview

- We analyze a Kyle model in which the strategic trader is a potential activist who can affect the value of a stock by exerting costly effort.
- From The Economist: Between 2010 and 2014, half the companies in the S&P 500 index had an activist shareholder and one in seven were the target of an activist campaign.
- We study the relation between market liquidity and activism.
- Main prior paper: Maug (JF, 1998), a single-period Kyle model with binary activism.
**Activism Model**

- Date $T$ of activism is fixed exogenously. Effort or lack of effort is publicly observed.

- One potential activist. Her blockholding is not publicly observed prior to $T$.

- The activist’s effort is chosen optimally, depending on her blockholding at $T$.

- The cost of the effort required to produce a share value of $v$ is $C(v) \geq 0$. We take $C(v) = \infty$ if $v$ is infeasible.

- Example (binary): $C(L) = 0$, $C(H) = c > 0$, $C(v) = \infty$ if $v \notin \{L, H\}$.

- Example (quadratic): $C(v) = (v - v_0)^2/(2\psi)$. 
Trading Model

- Prior to $T$, the potential activist can trade profitably on private information about her blockholding (and therefore private information about her intentions).
- After $T$, she has no private information and therefore cannot profitably trade. So, we assume trading ends at $T$.
- Trading is continuous during $[0, T]$ (for tractability). Follows Kyle (1985) model:
  - $X_t =$ large trader’s position. $X_0 \sim n(\mu_x, \sigma^2_x)$.
  - Noise trades $Z =$ Brownian motion with std dev $\sigma$.
  - Risk neutral, competitive market makers.
Value of Shares at $T$

- Value of $x$ shares to the activist at $T$ is
  \[ G(x) \overset{\text{def}}{=} \sup_v \{ vx - C(v) \} . \]

- $G$ is a convex function.

- Assume $C$ is lower semicontinuous and grows more than linearly:
  \[ \lim_{v \to -\infty, +\infty} \left| \frac{C(v)}{v} \right| = \infty \]

- Then there is an optimal $v$ for the activist. Let $V(x)$ denote an optimum.

- $V(x)$ is a subgradient of $G$ at $x$. Almost everywhere, $G'(x) = V(x)$ (the marginal value of shares to the activist is the market value – this is the envelope theorem).
Prices before $T$

- Define $Y_t = (X_t - X_0) + Z_t$. This is the aggregate order process observed by market makers and used by market makers to set prices.
- An equilibrium condition is that the price at each date $t < T$ equal the expected value of $V(X_T)$ conditional on the history of $Y$ until $t$.
- We look for an equilibrium in which the price at $t$ depends only on $Y_t$. Let $P(t, y)$ denote the price function.
Optimal Trading

- Assume the potential activist’s trades are of order $dt$ (always true in continuous-time Kyle models).
- So, $dX = \theta \, dt$ for some $\theta$.
- Information of the activist at $t$ is $X_0$ and history of $Z$ until $t$ (can infer $Z$ from prices). Therefore knows $Y_t = (X_t - X_0) + Z_t$.
- Value function of the potential activist is

$$J(t, x, y) = \sup_{\theta} \mathbb{E} \left[ G(X_T) - \int_t^T P(u, Y_u) \theta_u \, du \middle| X_t = x, Y_t = y \right]$$
HJB Equation

\[ 0 = \sup_\theta \left\{ -P \theta + J_t + J_x \theta + J_y \theta + \frac{1}{2} \sigma^2 J_{yy} \right\}. \]

Equivalently,

\[-P + J_x + J_y = 0, \]
\[ J_t + \frac{1}{2} \sigma^2 J_{yy} = 0. \]
Main Theorem

Define

$$\Lambda = 1 + \sqrt{1 + \frac{\sigma^2_x}{\sigma^2 T}}$$

The pricing rule

$$P(t, y) = E[V(\mu_x + \Lambda y + \Lambda(Z_T - Z_t))]$$ (1)

and trading strategy

$$\theta_t = \frac{1}{T - t} \left( \frac{X_t - \mu_x - \Lambda Y_t}{\Lambda - 2} \right)$$ (2)

constitute an equilibrium.
Main Theorem continued

- The share price post-activism is

\[ P(T, Y_T) = V(\mu_X + \Lambda Y_T) = G'(\mu_X + \Lambda Y_T) = G'(X_T) \]

- The distribution of \( Y \) given market makers’ information is that of a Brownian motion with zero drift and standard deviation \( \sigma \) (the same law as \( Z \)).

- The formula for \( P(t, Y_t) \) is the expected value of \( P(T, Y_T) \) conditional on \( Y_t \) and conditional on \( Y \) having the same law as \( Z \).

- Market makers view \( Y \) as being a Brownian motion with zero drift and the same std dev as \( Z \) because the potential activist’s trades are ‘inconspicuous’ (have zero mean) and because a continuous martingale \( Y \) with \( (dY)^2 = \sigma^2 dt \) is a Brownian motion with std dev \( \sigma \).
Main Theorem continued

The value function is

\[ J(t, x, y) = \frac{\Lambda - 1}{\Lambda} E \left[ G \left( \frac{\Lambda(x - Z_T) - \mu_x}{\Lambda - 1} \right) \right| Z_t = y \]

\[ + \frac{1}{\Lambda} E \left[ G(\mu_x + \Lambda Z_T) \right| Z_t = y \]. \]

The equilibrium price evolves as \( dP(t, Y_t) = \lambda(t, Y_t) \, dY_t \), where Kyle’s lambda is

\[ \lambda(t, y) = \frac{\partial P(t, y)}{\partial y}. \] (3)

Furthermore, \( \lambda(t, Y_t) \) is a martingale on \([0, T - \delta]\) for every \( \delta > 0 \), relative to the market makers’ information set.
Construction of the Value Function

- To construct the value function, we consider the hypothetical (non-equilibrium) strategy of not trading until just before $T$ and then trading until the price equals the marginal value. This value at $T$ is

$$J(T, x, y) \overset{\text{def}}{=} G(x) + \sup_{\bar{y}} G(x + \bar{y} - y) - \int_{y}^{\bar{y}} P(T, u) \, du$$

- The value at $t < T$ is the expectation of $J(T, x, Y_T)$ again viewing $Y$ as a Brownian motion with std dev $\sigma$. 
Lemma

Let $\varepsilon$ be a standard normal random variable that is independent of $Z$. Let $b$ be a nonnegative constant, and set $a = \sigma \sqrt{(2b + 1)T}$. Then, the solution $Y$ of the stochastic differential equation

$$dY_t = \frac{a\varepsilon - bZ_t - (b + 1)Y_t}{T - t} \, dt + dZ_t$$

on the time interval $[0, T)$ has the following properties:

- $Y$ is a Brownian motion with zero drift and standard deviation $\sigma$ on its own filtration on $[0, T]$.
- With probability 1,

$$Y_T = \frac{a\varepsilon - bZ_T}{b + 1}.$$

Remark: The case $b = 0$ is a Brownian bridge.
Comparison to the Standard Kyle Model

- In the standard Kyle model,
  - $\varepsilon = N^{-1}(F(\nu))$ where $N$ is the standard normal cdf, and $F$ is the cdf of $\nu$.
  - $a = \sigma \sqrt{T}$
  - $b = 0$
  - $X_T = X_0 + \sigma \sqrt{T} N^{-1}(F(\nu)) - Z_T$

- In our model,
  - $\varepsilon = (X_0 - \mu_x)/\sigma_x$
  - $a = \sigma_x/(\Lambda - 2)$
  - $b = 1/(\Lambda - 2)$
  - $X_T = \mu_x + \frac{\Lambda}{\Lambda-1}(X_0 - \mu_x - Z_T) = \mu_x + \Lambda Y_T$
Efficiency

- \( P(0, 0) = E[V(T, Y_T)] \) reflects the value per share expected to be created by activism.
- We measure economic efficiency by \( P(0, 0) \).
- We should subtract the cost of activism, but we assume it is small on a per-share basis (activists cover costs from relatively small percentage shareholdings).
Market Liquidity

- We measure market illiquidity by the expected average Kyle’s lambda.
- Because lambda is a martingale,

\[ \lambda(0, 0) = \mathbb{E} \int_0^T \lambda(t, Y_t) \, dt \]

- So, we measure illiquidity by \( \lambda(0, 0) \).
Comparative Statics

- Let $\bar{P}$ denote $P(0, 0)$ as a function of model parameters. Let $\bar{\lambda}$ denote $\lambda(0, 0)$ as a function of model parameters.
- We are interested in the comparative statics:

$$\frac{\partial \bar{P}}{\partial \sigma}, \quad \frac{\partial \bar{\lambda}}{\partial \sigma}$$

and the same with respect to $\sigma_x$, $\mu_x$, and parameters of the cost function $C(v)$.
- We have

$$\bar{P} = E[V(\mu_x + \Lambda Y_T)], \quad \bar{\lambda} = \Lambda E[V'(\mu_x + \Lambda Y_T)]$$
Comparative Statics

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- We have
  \[
  \bar{P} = \mathbb{E}[V(\mu_x + \Lambda Y_T)], \quad \bar{\lambda} = \Lambda \mathbb{E}[V'(\mu_x + \Lambda Y_T)]
  \]
Comparative Statics: Noise Trading

- $\frac{\partial \bar{P}}{\partial \sigma} \geq 0$ if $V$ is convex and $\leq 0$ if $V$ is concave, because an increase in $\sigma$ is a mean-preserving spread in $\mu_x + \Lambda Y_T$.
- The effect of $\sigma$ on market liquidity is ambiguous.
- We give an example in which $V$ is affine, other examples in which $V$ is strictly convex, and one example in which $V$ is neither convex nor concave.
- A concave $V$ must be unbounded below, which means that unbounded value destruction is possible.
1. Quadratic Cost

- $C(v) = \frac{(v - v_0)^2}{2\psi}$
- Higher $\psi$ means more productive (less cost)
- Activist can destroy value as well as create value
- $V(x) = v_0 + \psi x$ is convex and concave.
- Kyle’s lambda is constant over time.
- An increase in $\sigma$ increases market liquidity but has no effect on efficiency.
- An increase in $\psi$ reduces market liquidity and increases (reduces) efficiency if $\mu_x > 0$ ($\mu_x < 0$)
2. Asymmetric Quadratic Cost

- Quadratic for $v > v_0$ and $\infty$ for $v < v_0$
- $V(x) = v_0 + \psi x^+$
- An increase in $\sigma$ increases market liquidity and increases efficiency
- Increases in other parameters reduce liquidity and increase efficiency.
Examples 1 and 2 when Noise Traders Buy
Examples 1 and 2 when Noise Traders Sell
4. Binary

- \( C(\nu_0) = 0, \quad C(\nu_0 + \Delta) = c, \quad C(\nu) = \infty \) otherwise.
- \( V(x) = \nu_0 \) if \( x\Delta < c \), \( V(x) = \nu_0 + \Delta \) if \( x\Delta > c \)
- An increase in \( \sigma \) increases efficiency if \( \mu_x \Delta > c \) and reduces efficiency if \( \mu_x \Delta < c \) (Maug, 1998)
- An increase in \( \sigma \) can either increase or reduce market liquidity.
- Measure productivity by \( \Delta \) and \( \psi \overset{\text{def}}{=} \Delta/c \). An increase in either reduces market liquidity and increases efficiency.
Binary Example

Effects of an increase in liquidity trading $\sigma$ on efficiency $\bar{P}$ and market liquidity $1/\bar{\lambda}$
Conclusion

- Market liquidity and activism are both endogenous. The cross-sectional relation between them depends on the source of cross-sectional variation and on the activism technology.
- Under a natural convexity assumption, an increase in noise trading increases activism. But it may reduce market liquidity.
- An increase in activist productivity generally increases efficiency and reduces market liquidity.