Continuous Time Finance and Mean-Variance in Post Retirement Planning

Presented at Conference in Honor of Steven's 65th Birthday

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Introduction

- *"Society needs scientists willing to explain the content and consequence of their work to the public. These are modern Renaissance men and women…who understand science beyond a superficial level."*
- ~ Steven

About GuidedChoice

- GuidedChoice mission: Financial freedom for all
- Provide confidence and peace of mind from real financial facts and reliable projections
- GuidedSavings℠ GuidedChoiceIRA℠ GuidedSpending℠

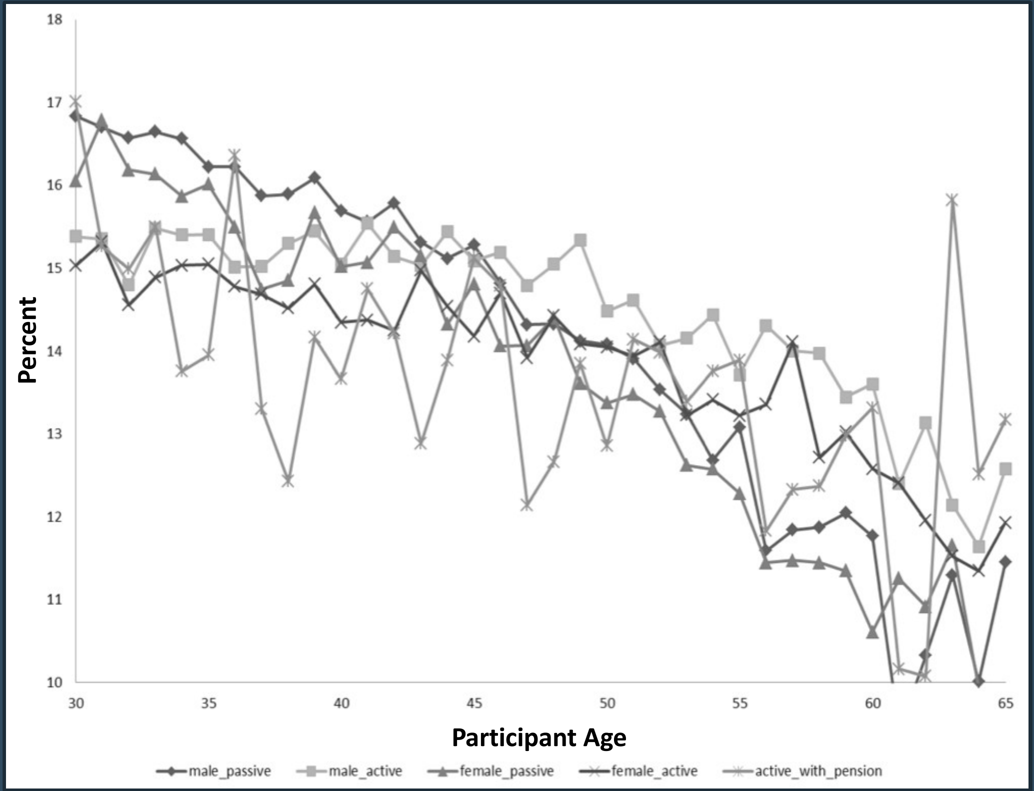
Current 401(k) Market

$ 5 trillion dollars

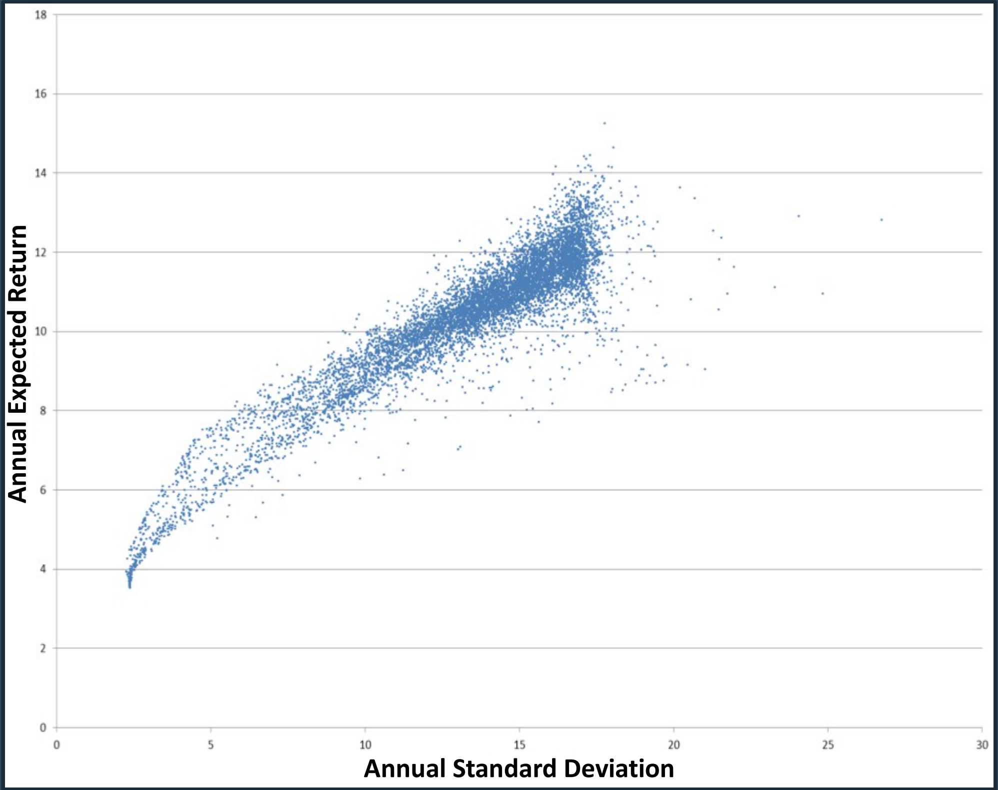
70 million participants

- A — Half of participants outsource their asset allocation to target date funds
- B — Women tend to take less risk
- C — Those with supplemental pension plans invest more conservatively than those without
- D — Young participants are not taking enough risk
- E — Naïve diversification

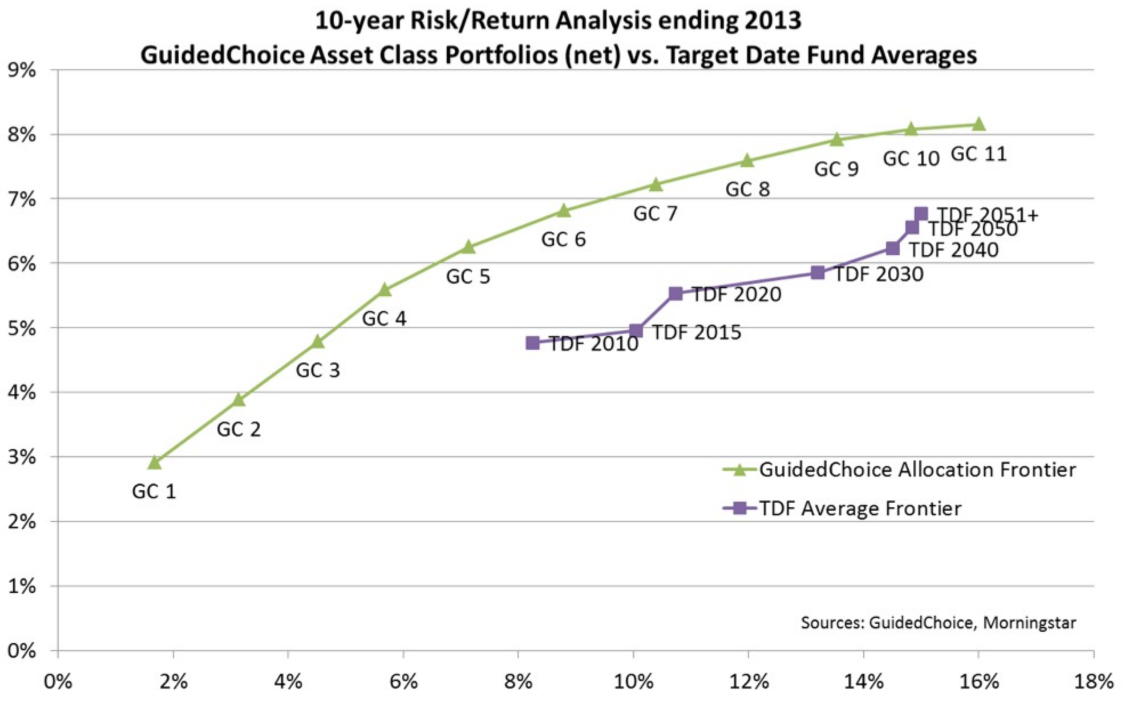
Total Risk Distribution



Mean-Variance Distribution: 1989-2013 Returns



Performance

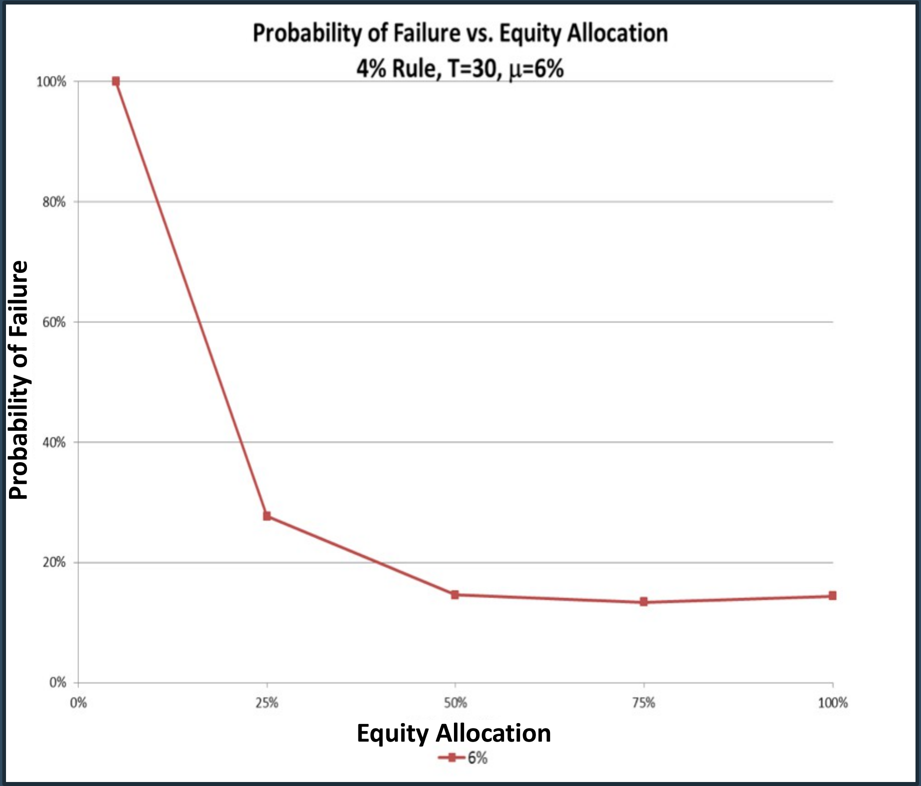


GC performance reflects GuidedChoice asset allocations to representative index returns, rebalanced monthly, and are net of an additional 60bp, accrued quarterly

Spending Problem : Practitioner’s Solution

- 4% Rule: Withdraw constant dollars equal to 4% initial wealth
- $dX_t = \mu\pi_t X_t dt - C * dt + \sigma\pi_t X_t dw_t$
- Probability($X_{30} \leq 0) \leq 0.06$
- Historically with $\pi = 0.5$
- An acceptable and good solution

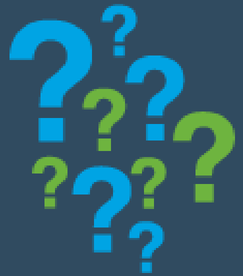
Other Market Condition



Spending Problem : Mathematician's Solution

- $dp_t = p_t(\mu dt + \sigma dw_t)$
- $E \int_0^\tau e^{-\beta t} U(c_t) dt$
- $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$
- $c_s^* = \frac{1}{A(s)} X_s, A(s) \stackrel{\text{def}}{=} \int_s^T e^{-k(\gamma)(T-t)}\, dt$
- $k(\gamma) = \frac{\beta}{\gamma} + r\left(1 - \frac{1}{\gamma}\right) + 0.5 * \theta^2 * \frac{1}{\gamma} * (1 - \frac{1}{\gamma})$
- $\pi_s^* = \frac{1}{\gamma}\frac{\mu - r}{\sigma^2} = \frac{1}{\gamma}\frac{\breve{\mu}}{\sigma^2} = \frac{\theta}{\gamma\sigma}$ (Merton Line)

Practical Questions



What is my risk aversion parameter?

What is my utility discount rate?

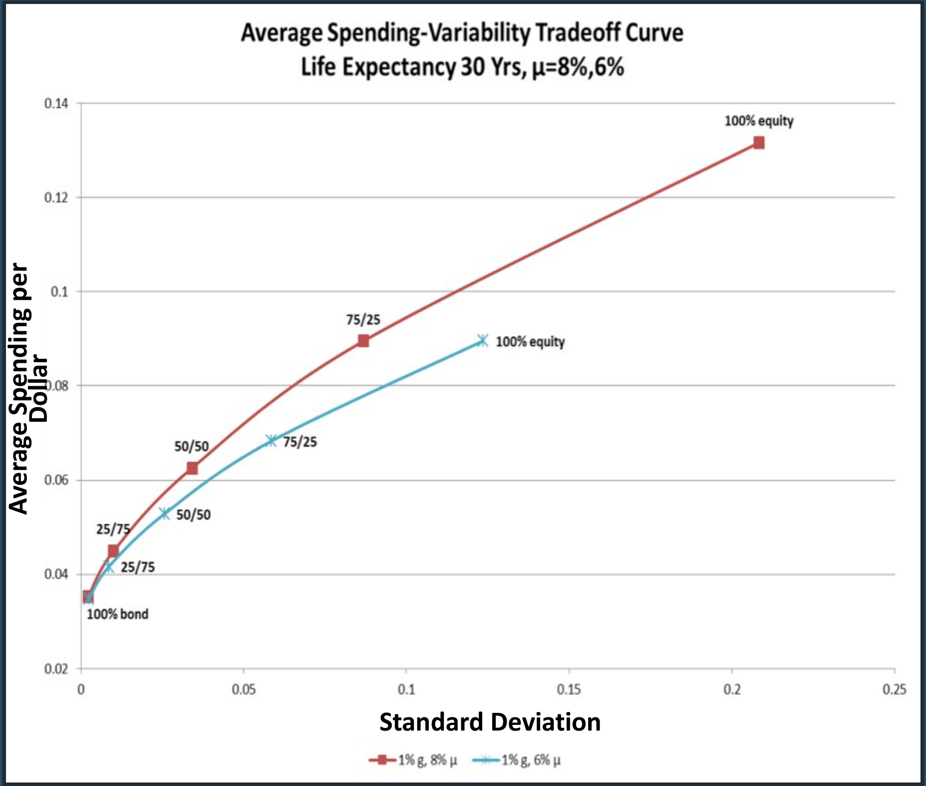
Typical question: If offered the choices listed below, which option would you select?

- Accept $1M now
- Play a game where you have a 90% chance of winning $2M and a 10% chance of winning nothing
- Play a game where you have a 50% chance of winning $5M and a 50% chance of winning nothing

Experiment's Solution

- For any annuitization rate/portfolio policy (g, T, π),
- $AF(t) \stackrel{\text{def}}{=} \frac{1}{\int_t^T e^{-g(T-s)} ds} = \frac{g}{1-e^{-g(T-t)}}$
- $c_t = AF(t) * X_t$
- $c_t = x * AF(0) * e^{(\pi\mu-g)t-0.5*(\pi\sigma)^2 t+\pi\sigma W_t}$

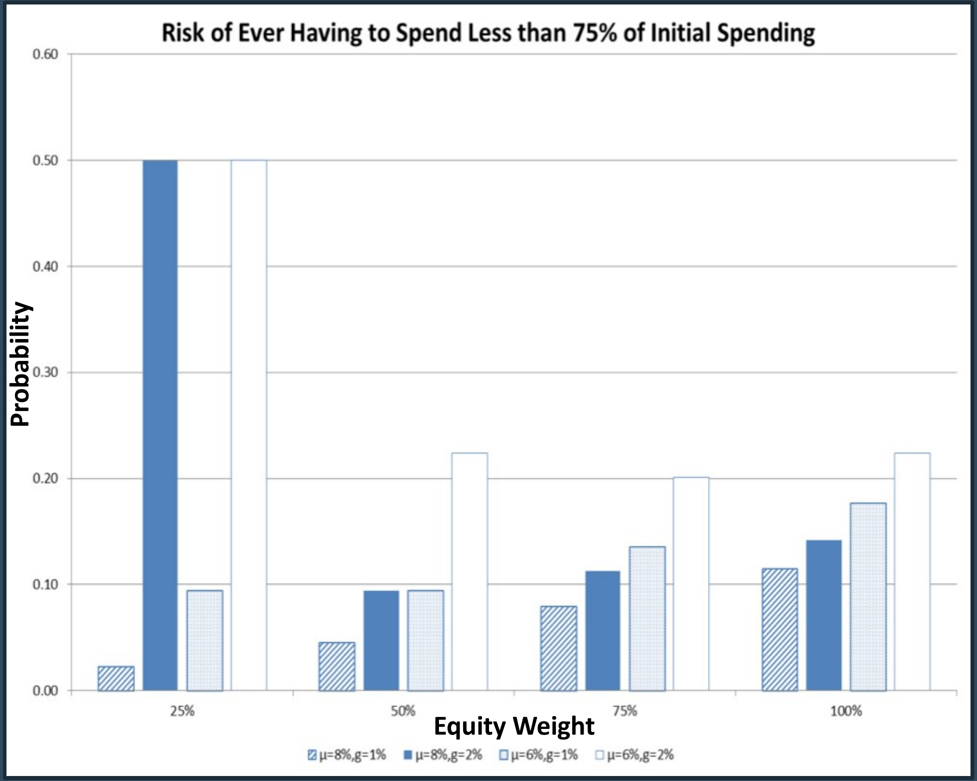
Mean-Variance Analysis



Probability of Spending Less

- $P(c_t < \kappa * x * AF(0))$
- $=P\left(e^{(\pi\mu-g)t-0.5*(\pi\sigma)^2 t+\pi\sigma W_t} < \kappa\right)$
- $=1-\Phi\left(\frac{-\ln(\kappa)+(\pi\mu-g)t-0.5*(\pi\sigma)^2 t}{\pi\sigma\sqrt{t}}\right)$
- $max_{t\geq 0} P(c_t < \kappa * x * AF(0))$

Probability of Spending Less



More Interesting Problem

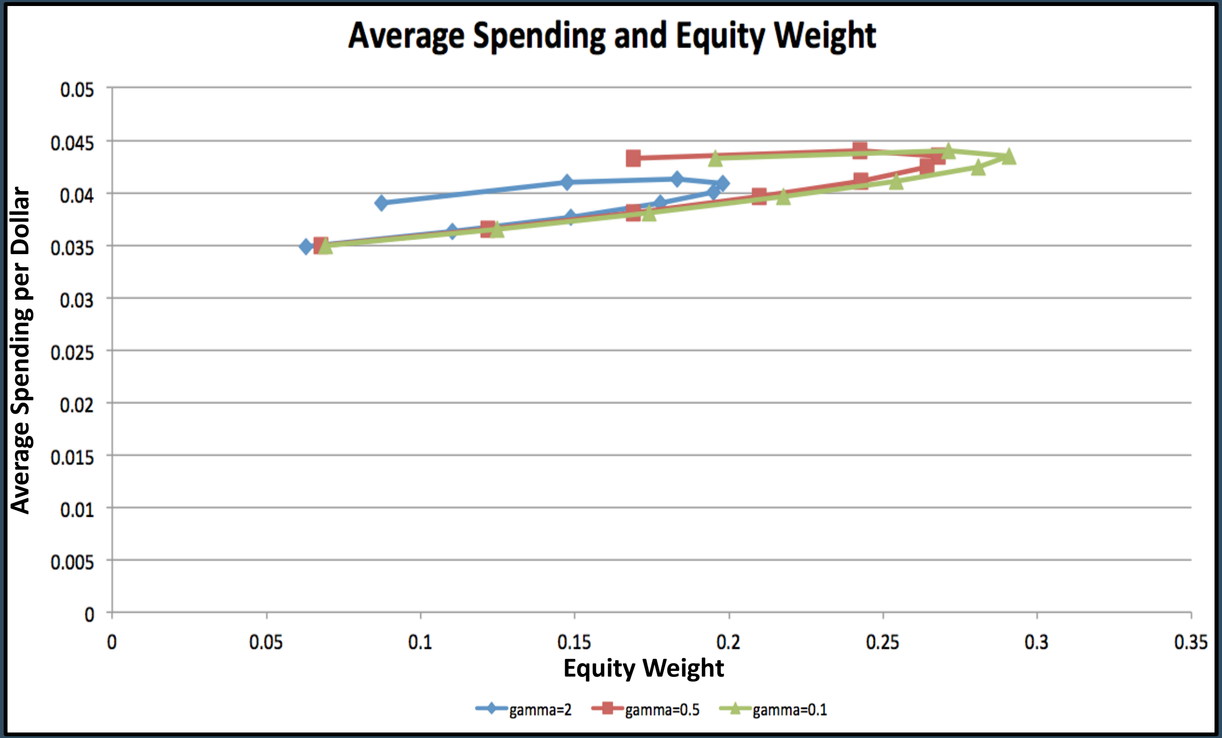
- Budget constraints: $\underline{c} \leq c_s \leq \overline{c}, 0 \leq s \leq T$
- No bankruptcy implies:

  Initial Wealth x >= Present Value($\underline{c}$)
- Initial Wealth x >= Present Value($\overline{c}$)
- No analytical solution for $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$

Portfolio Insurance and Profit-Taking

- $U(c) = -c * (\bar{c} - \underline{c})^{-\gamma} + \frac{(c-\underline{c})^{1-\gamma}}{1-\gamma}$
- $I(y) = \underline{c} + (y + (\Delta c)^{-\gamma})^{-\frac{1}{\gamma}}, \forall\, y \geq 0$
- $c_s = \underline{c} + (\Delta c)(\lambda(t,x) * \Delta c^{\gamma} * \zeta(t,s) + 1)^{-\frac{1}{\gamma}}$
- $E \int_t^T \zeta(t,\tau)\,(\lambda(t,x)\zeta(t,\tau) + (\Delta c)^{-\gamma})^{-\frac{1}{\gamma}} d\tau = x - \underline{c} * (T - t)$

Selection of Risk Aversion Parameter



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THANK YOU

& Happy Birthday Steven!

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