Benchmarks in Over-the-Counter Markets

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Common Over-the-Counter Market Benchmarks

- LIBOR, EURIBOR, TIBOR, ...
- SONIA, EONIA, ...
- WM/Reuters foreign exchange fixings.
- Gold, Silver, Palladium, Platinum, ...
- Oil (Brent, WTI), Natural Gas, Iron Ore (IODEX), ...
- Pharmaceuticals (Average Wholesale Price).
Key Benchmark Functions

1. Contractibility for price-contingent claims.


3. Pre-trade price transparency: allowing easier comparison shopping in OTC markets.
Welfare Roles of Benchmarks in Search Markets

1. Increasing the volume of beneficial trade through:
   - Signaling when there are high gains from trade.
   - Improving the share of gains offered to traders.

2. Reducing total search costs.

3. Facilitating more efficient trade matching between dealers and customers, through:
   - Improving the ability of traders to detect when quotes are from high-cost dealers.
   - The use of benchmarks by low-cost dealers as a “price transparency weapon.”
## Selected LIBOR and EURIBOR Dependencies

(amounts in billions of USD equivalent notional)

<table>
<thead>
<tr>
<th>Category</th>
<th>U.S.</th>
<th>LIBOR fraction</th>
<th>Eurozone</th>
<th>EURIBOR fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Syndicated loans</td>
<td>3400</td>
<td>97%</td>
<td>535</td>
<td>90%</td>
</tr>
<tr>
<td>Bilateral corporate loans</td>
<td>1650</td>
<td>≈40%</td>
<td>4322</td>
<td>60%</td>
</tr>
<tr>
<td>Retail mortgages</td>
<td>9608</td>
<td>15%</td>
<td>5073</td>
<td>28%</td>
</tr>
<tr>
<td>Floating rate notes</td>
<td>1470</td>
<td>84%</td>
<td>2645</td>
<td>70%</td>
</tr>
<tr>
<td>Interest rate swaps</td>
<td>106700</td>
<td>65%</td>
<td>137553</td>
<td>high</td>
</tr>
<tr>
<td>Exchange-traded derivatives</td>
<td>32900</td>
<td>93%</td>
<td>17300</td>
<td>100%</td>
</tr>
</tbody>
</table>

Legacy IBOR fixing method

- B1
- B2
- B3

Loan: B1 to B2, B2 to B3, B3 to B1
Report: B1 to benchmark fixing administrator, B2 to benchmark fixing administrator, B3 to benchmark fixing administrator
Fixing: Benchmark fixing administrator to B1, B2, B3

Diagram: A network of three nodes (B1, B2, B3) connected by loan and report arrows, with a connection to a benchmark fixing administrator.
distance \( k \) from the present gets larger. Thus the algorithm includes noncontemporaneous data to compensate for the low density of transactions on any given day, but downweights the older data in light of its staleness (Duffie, Skeie, and Vickrey 2013).

The results of this exercise are plotted in Figure 2, which compares the constructed LIBOR+ to actual LIBOR for each of the 1-, 3- and 6-month tenors. As can be seen, while LIBOR+ is always more volatile on a day-to-day basis than LIBOR—which is not surprising given the opinion-based nature of LIBOR—the levels of the two time series track each other reasonably closely at both the 1-month and 3-month tenors. At the 6-month tenor, the fit is considerably less good. Some of this deterioration in fit is due to the paucity of transactions at 6-month terms. But some of it is due to a particular form of sample selection—the fact that during a period of market stress, only the highest credit-quality banks find it economically sensible to issue at a 6-month maturity. This selection effect tends to make the transactions-based LIBOR+ lower than the judgmentally reported LIBOR during stressful periods in the banking sector. Nevertheless, given that the vast majority of contracts in dollar LIBOR reference the 1- and 3-month tenors, the LIBOR+ methodology holds considerable practical promise, especially if the data supporting it can eventually be augmented to capture the entire universe of certificate-of-deposit and commercial paper transactions.

However, even if a transactions-based LIBOR+ methodology can be made to work well from an economic perspective, there remains the crucial question of whether it also “works” legally. In other words, for the large stock of existing legacy contracts that reference LIBOR, is it possible to seamlessly substitute a fixing along the lines of LIBOR+ without causing private litigants to challenge this substitution? We will return to this question later.

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### Table 2

Transactions Data on Unsecured Bank Borrowing

<table>
<thead>
<tr>
<th></th>
<th>Number of Trades</th>
<th>Numbers of Issuers</th>
<th>Volume ($mn)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>O/N 1W 1M 3M 6M</td>
<td>O/N 1W 1M 3M 6M</td>
<td>O/N 1W 1M 3M 6M</td>
</tr>
<tr>
<td><strong>Daily Avg</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2014</td>
<td>469 74 21 19 18</td>
<td>15 9 7 8 8</td>
<td>20,223 3,204 888 706 718</td>
</tr>
<tr>
<td>2013</td>
<td>511 95 18 25 13</td>
<td>16 9 6 8 8</td>
<td>52,312 4,157 702 1,066 474</td>
</tr>
<tr>
<td>2012</td>
<td>344 62 24 31 13</td>
<td>17 10 8 9 9</td>
<td>4,889 2,637 888 1,211 452</td>
</tr>
<tr>
<td>2011</td>
<td>435 79 38 34 18</td>
<td>21 15 14 11 11</td>
<td>18,945 3,356 1,407 1,381 706</td>
</tr>
<tr>
<td><strong>Daily Max</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2014</td>
<td>538 127 42 45 40</td>
<td>17 13 10 12 12</td>
<td>23,853 5,460 1,869 1,903 1,881</td>
</tr>
<tr>
<td>2013</td>
<td>878 280 78 126 76</td>
<td>20 18 13 17 17</td>
<td>38,722 13,043 3,479 5,904 2,892</td>
</tr>
<tr>
<td>2012</td>
<td>521 225 80 112 55</td>
<td>24 20 19 19 19</td>
<td>23,985 10,007 3,613 4,539 2,146</td>
</tr>
<tr>
<td><strong>Daily Min</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2014</td>
<td>406 31 3 8 2</td>
<td>14 5 3 4 2</td>
<td>16,986 1,279 77 222 50</td>
</tr>
<tr>
<td>2013</td>
<td>187 7 1 1 1</td>
<td>10 3 1 1 1</td>
<td>6,910 204 5 1 1</td>
</tr>
<tr>
<td>2012</td>
<td>32 4 0 0 0</td>
<td>7 2 0 0 0</td>
<td>1,299 124 0 0 0</td>
</tr>
<tr>
<td>2011</td>
<td>235 10 3 3 0</td>
<td>17 4 1 1 1</td>
<td>9,608 242 75 24 0</td>
</tr>
</tbody>
</table>

**Source:** Market Participants Group, Final Report, March 2014.

**Notes:** This table displays daily average, maxima, and minima for number of trades, number of issuers, and dollar volume of unsecured bank borrowing transactions in the commercial paper (CP) and certificate-of-deposit (CD) markets based on a sample from a unit of J. P. Morgan over the period 2011 through January 2014. Maturity buckets are defined as follows: O/N = 1 day to 4 days, 1W = 6 days to 8 days, 1M = 28 days to 32 days, 3M = 85 days to 95 days, 6M = 175 days to 185 days. "$mn$" means “millions of dollars.”
What is a Suitable Riskless Interest Rate Benchmark?

Despite the potential promise of LIBOR+ for certain bank-based transactions, we believe that it would be a mistake for such a benchmark to shoulder the burden of being the primary reference rate for the entire interest-rate derivatives market. To understand why, compare the magnitudes in Tables 1 and 2. At the commonly
A fixing “anchored in transactions”

IOSCO Principle 7
Benchmark = \sum_i w_i p_i
Volume weighted average price (VWAP)

\[ w_i = f(s_i) \]

size weighting \( f(s) \)
Nonlinear size weights

trade price

trade size

size weighting f(s)
Eliminate or underweight price outliers

\[ w_i = 0 \]
Bank CP-CD primary market
Treasury – Eurodollar Spread (percent)

Data: Federal Reserve Bank of St. Louis
Proposed Reform of Interbank Offered Rates
Recommendations of MPG, OSSG, Duffie-Stein

1. New transactions-based fixing method for IBORs, called “IBOR+”
   - Capture transactions on all wholesale unsecured bank borrowings near relevant tenor.
   - Include several days of lagged transactions, with declining weights.
   - Seamless transition for at least 1-month and 3-month USD LIBOR.

2. Introduce new benchmarks for rates trading applications.
   - Treasury bill rate.
   - OIS
   - Compound 1-day treasury repo rates to get 3-month rate.
Using covered interest parity

A

Currency X rate

B

Currency Y rate

Forward FX rate
Using put-call parity

\[ \text{Bond} = \text{Underlying} - \text{Call} + \text{Put} \]
If you build it, will they come?
Many short rates are highly correlated

Data: Federal Reserve Bank of St. Louis
90-day Overnight Index Swap (OIS) rate $R$

$\left(1 + r_1\right) \left(1 + r_2\right) \left(1 + r_3\right) \left(1 + r_4\right) \ldots \left(1 + r_{90}\right)$

Days:

- Fixed payment $1 + R$
- Floating payment

$\text{days}$
Loan costs and revenues

spread over IBOR

IBOR

cost of funds

gross margin
Related Work on Search Market Transparency

- Benabou and Gertner (1993) analyze the influence of inflationary uncertainty on welfare and the split of surplus between consumers and two firms.


Dealers Post Quotes on Platforms

The cost of dealer $i$ is $c_i = c + \epsilon_i$, where $c$ is common, $\epsilon_i$ is idiosyncratic.

There is a benchmark if the common cost component $c$ is published.

The quote $p_i$ of dealer $i$ has an equilibrium probability distribution $F$ that depends on $c$ and $\epsilon_i$, and whether there is a benchmark.

The payoff of dealer $i$ is $(p_i - c_i)Q_i$, where $Q_i$ is the total volume of trades.
Fast Traders Pick the Minimum Offer

All traders value the asset at trader at some constant value $v$.

A fraction $\mu$ of traders are “fast,” that is, have no search cost.

In this example, the payoff of the fast trader is $v - 1.7$. 
Feasible Search Path of an Entering Slow Trader

Slow traders visit dealer trade platforms in random order, facing a Pandora Problem.

The net payoff of this path is $v - 1.9 - 3s$
Outline of Results

- A welfare comparison of market equilibria with and without a benchmark.

- With heterogeneous-cost dealers, how benchmarks improve matching efficiency (and welfare).

- The incentives of homogeneous-cost dealers to introduce a benchmark.

- The strategic introduction of benchmarks by low-cost dealers to increase market share.

- Benchmark manipulation incentives for dealers.
Equilibrium Search of a Slow Trader with a Benchmark

Enter with a probability $\lambda_c$ that depends on the observed benchmark $c$.

Immediately accept the first offer below an optimal reservation price $r_c$.

The net payoff of this path is $v - 1.9 - 2s$. 

\[ \text{enter (s)} \rightarrow 2.1 \rightarrow 1.9 \rightarrow 2.2 \rightarrow 1.7 \rightarrow 2.3 \]
The support of the distribution of $c$ is $[c, \bar{c}]$.

We begin with the case of only low-cost dealers.

We examine behavior on the event $\{c < v - s\}$. (Otherwise, slow traders don’t enter and dealers compete à la Bertrand, offering to sell for $c$.)

The unique equilibrium probability distribution $F$ of offer quotes has no atoms and has upper support limit $r_c$. 
Dealer Quote Strategy

For a dealer, the probability that a quote-observing trader is fast is

\[ q(\lambda_c) = \frac{\mu}{\mu + \frac{1}{N} \lambda_c (1 - \mu)}. \]

Dealers are indifferent between all price offers in the support of \( F \), so

\[ [1 - q(\lambda_c) + q(\lambda_c) \left( 1 - F(p)^{N-1} \right)] (p - c) = [1 - q(\lambda_c)] (r_c - c). \]

Solving,

\[ F(p) = 1 - \left[ \frac{\lambda_c (1 - \mu)}{N \mu} \frac{r_c - p}{p - c} \right]^{\frac{1}{N-1}}. \]
Slow Trader Strategy

Pandora solution of Weitzman (1979): Indifference to search when observing the quote $r_c$ implies that

$$v - r_c = v - s - \mathbb{E}_F(p).$$

Solving,

$$r_c = c + \frac{1}{1 - \alpha(\lambda_c)} s,$$

where

$$\alpha(\lambda_c) = \int_0^1 \left(1 + \frac{N\mu}{\lambda_c(1 - \mu)} z^{N-1}\right)^{-1} dz < 1.$$

An interior entry probability $\lambda_c$ solves

$$s = (1 - \alpha(\lambda_c))(v - c).$$
Enter with probability \( \lambda \).

Accept the offer on the first platform visited if it is below \( v \).

Then exit.

Because \( v < 2.1 \), this path has net payoff \(-s\).
When Does a Benchmark Improve Welfare?

- Change variables to gain from trade \( x = \max(v - c, 0) \).

- Letting \( \Lambda(x) = \lambda_c \), the social surplus with a benchmark is
  \[
  W(x) = \mu x + \Lambda(x)(1 - \mu)(x - s).
  \]

- The total social surplus with no benchmark is \( W[\mathbb{E}(x)] \).

- If \( \mu \) is small enough or \( s \) is at least a given fraction of \( X \), then \( W(\cdot) \) is sub-differentiable at \( \mathbb{E}(x) \).

- In these cases, \( \mathbb{E}[W(x)] \geq W[\mathbb{E}(x)] \), so benchmarks increase expected social surplus.
Gain from trade $x$ 

$$W_b(x)$$

Supporting hyperplane at $X$

$$x - (1 - \mu)s$$

$$\frac{s}{1-\alpha}$$

Surplus $s$
If the expected gain from trade of slow traders is sufficiently large relative to search costs, then even without the benchmark all of the slow traders may enter.

In the presence of the benchmark, however, slow-trader entry may be low in the event of a high realization of $c$ (still allowing gains from trade).

Thus, adding a benchmark could reduce welfare if the entry of slow traders is already nearly efficient without the benchmark.
Matching Efficiency

Proposition. Suppose the search cost is sufficiently low and there is always a gain from trade \((v > \bar{c} + \Delta)\). Then, with a benchmark:

- All trade is with low-cost dealers.

- If, in addition, the search cost is not too low, then slow traders always trade with the first encountered low-cost dealer.

Theorem. If the search cost is within a specified interval and if \(\bar{c} > c + \Delta\), then the expected social surplus is strictly greater in the equilibrium with a benchmark than in any equilibrium without a benchmark.
Incentive for Dealers to Introduce a Benchmark

**Theorem.** Suppose all dealers have the same cost, and the search cost is high enough. Then dealer profits are higher with a benchmark than without.

Whenever dealers would opt for the benchmark in this sense, it must be the case that the introduction of the benchmark raises social surplus. The converse is not true.
Low-Cost Dealers Can Use Benchmarks Strategically

- A slight change in the cost distribution, so that the number $L$ of low-cost dealers is zero or at least two.

- Any non-trivial coalition of dealers can commit to a benchmark (by voting).

- Dealers enter if and only if their expected profit is strictly positive.

- The number of entering dealers is publicly observed.
Proposition. Suppose that the dealer cost difference $\Delta$ is sufficiently large and the search cost $s$ is not too high. Then:

- There exists an equilibrium of the extended game in which low-cost dealers always vote in favor of the benchmark, and high-cost dealers always vote against it. Moreover, there are no profitable group deviations in the voting stage.

- If the environment is competitive (that is, $L \geq 2$), the benchmark is introduced, all high-cost dealers stay out of the market, all low-cost dealers enter the market, and all traders enter the market.

- If the environment is uncompetitive ($L = 0$), the benchmark is not introduced, and all dealers enter the market.
Manipulation Incentives

▶ Suppose homogeneous dealer costs.

▶ If traders ignore the potential of manipulation and dealers can arrange for any benchmark distortion at no cost, then dealers would falsely announce that the benchmark is \( v - s/(1 - \alpha(1)) \).

▶ A mechanism is a pair \((M, g)\), where \( M = (M_1 \times \cdots \times M_N) \) is the product of the message spaces of the \( N \) respective dealers, and where \( g : M \to [\underline{c}, \bar{c}] \times \mathbb{R}^N \).

▶ The function \( g \) maps the dealers’ messages \((m_1, \ldots, m_N)\) to an announced benchmark \( \tilde{c} \) and to transfers \( t_1, \ldots, t_N \)

**Proposition.** Truthful revelation of \( c \) is Nash implementable, but is not fully Nash-implementable.