Happy Birthday Bela

Random Graphs 1985

The cover time of random walks on random graphs

Colin Cooper
Alan Frieze

The cover time of random walks on random graphs

Colin Cooper
Alan Frieze
and
Eyal Lubetzky

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$(1 - o(1))n \ln n \leq C_G \leq (1 + o(1))\frac{4}{27}n^3$: Feige (1995)

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- The cover time of random geometric graphs.
- The cover time of random graphs with a fixed degree sequence.

Cover time of $G = G_{n,p}$.
Jonasson (1998) proved:

- If $\frac{np}{\ln n} \to \infty$ then $C_G = (1 + o(1))n \ln n$ **whp**.
- If $\frac{np}{\ln n} \to c$, c constant, then **whp** $C_G \geq A_c n \ln n$ for some constant A_c.

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Cooper and Frieze (2003)

If $d = c \ln n$ where $(c - 1) \ln n \to \infty$ then **whp**

$$C_G \sim c \ln \left(\frac{c}{c-1} \right) n \ln n.$$

Note that **whp** $G_{n,p}$ is connected here.

Cover time of giant component

Suppose now that $np = d > 1$.

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If x is the unique solution in $(0,1)$ of $x = 1 - e^{-dx}$ then **whp** K_g has xn vertices and $dx(2-x)n/2$ edges.

If $1 < d = o(\ln n)$ then **whp**

$$\begin{aligned} C_g &\sim \frac{dx(2-x)}{4(dx - \ln d)} n(\ln n)^2 \\ &\sim \frac{1}{4} n(\ln n)^2 \qquad \text{if } d \to \infty. \end{aligned}$$

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If $d \sim \alpha \ln n$ where $0 < \alpha < 1$ is constant then **whp**

$$C_g \sim \gamma n (\ln n)^2$$

where

$$\gamma = \max \{ \alpha\ell(1 - \alpha\ell) : \ \ell \textit{ is a positive integer} \}$$

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Cooper and Frieze (2006)

If $d = (1-\delta)\ln n$ where $\delta = o(1)$ and $\delta \ln n \leq \ln\ln n$ then **whp**

$$C_g \sim (\ln\ln n + \max\{\delta, 0\}) n \ln n.$$

Note that if $\delta \ln n \to +\infty$ then **whp** $G_{n,p}$ is connected.

Cover time of Regular Graphs
Cooper and Frieze (2005)

Suppose that $r \geq 3$ and $G = G_{n,r}$ denotes a random r-regular graph with vertex set $[n]$. Then **whp** its cover time satisfies

$$C_G \sim \frac{r-1}{r-2} n \ln n.$$

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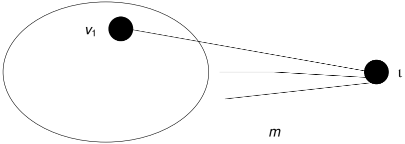
More generally, if $C_G^{(k)}$ is the time to get within $k = O(1)$ of every vertex then

$$C_G^{(k)} \sim \frac{1}{(r-2)(r-1)^{k-1}} \, n \ln n.$$

Cover time of preferential attachment graph
Cooper and Frieze (2007)

Sequence of random graphs $G(t)$
$G(t) = G(t-1)$ plus vertex t and m random edges $\{t, v_i\}, i = 1, 2, \ldots, m$.

The vertices $v_1, v_2, \ldots, v_m$ are chosen with probability proportional to their degree after step $t-1$.



Whp

$$C_G \sim \frac{2m}{m-1} t \ln t, \qquad \textit{for } m \geq 2.$$

Cover time of random digraphs
Cooper and Frieze (2012)

If $d = c \ln n$ where $c - 1$ is at least a positive constant then **whp**

$$C_D \sim c \ln \left(\frac{c}{c-1} \right) n \ln n.$$

Note that **whp** $D_{n,p}$ is strongly connected here.

Random graphs with a fixed degree sequence.
Abdullah, Cooper and Frieze (2012): $\delta \geq 3$
Cooper, Frieze and Lubetzky (20??): $\delta \geq 2$

Suppose that

$$2 \leq d_1 \leq d_2 \leq \cdots \leq d_n \leq N^{\zeta_0} \text{ where } \zeta_0 = o(1).$$

where N is the number of vertices of degree at least three.

Let $M = O(N)$ be the number of edges incident with vertices of degree at least three.

Let ν_2 be the number of vertices of degree two and let

$$\xi = \frac{M}{\nu_2 + M}$$

We use the following model for the random graph $G_{\mathbf{d}}$:

- Build the kernel K: The random graph with degree sequence $\mathbf{d}_{\geq 3}$.
- Sprinkle the ν_2 vertices of degree two randomly onto the edges of K.

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Theorem

If $G = G_{\mathbf{d}}$ and d is the minimum degree in K then w.h.p.

$$C_G \sim \begin{cases} \frac{2(d-1)}{d(d-2)} M \ln M & \text{if } \nu_2 = M^{o(1)}. \\ \psi_{\alpha,d} M \ln M & \nu_2 = M^{\alpha} \text{ where } 0 < \alpha < 1 \text{ is constant.} \\ \frac{(M+\nu_2)\ln^2 M}{-8\ln(1-\xi)} & \text{if } \nu_2 = \Omega(M^{1-o(1)}) \end{cases}$$

Here $\psi_{\alpha,d}$ is some explicitly given function.

If $p = \frac{1+\epsilon}{n}$ where $\epsilon = o(1)$ and $\epsilon^3 n \to \infty$ then w.h.p. $G_{n,p}$ has a unique giant component C_1 with a 2-core C_2. Our theorem applies to C_2.

We can model C_2 as $G_{\mathbf{d}}$ where K has $M \sim 2\epsilon^3 n$ and $\nu_2 \sim 2\epsilon^2 n$, Ding, Kim, Lubetzky and Peres (2011). So, w.h.p., if $G = C_2$,

$$C_G \sim \frac{\epsilon}{4} n \log^2(\epsilon^3 n).$$

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We were hoping to analyse the cover time of C_1 in this range. Based on our earlier results on the giant, we conjecture that if $G = C_1$ then w.h.p.

$$C_G \sim n \log^2(\epsilon^3 n)$$

First Visit Time Lemma.

Suppose that the connected graph $G = (V, E)$ has n vertices and m edges.

(For digraphs we need strong connectivity).

First Visit Time Lemma.

Suppose that the connected graph $G = (V, E)$ has n vertices and m edges.

Let $\pi_x = \frac{deg(x)}{2m}$ denote the steady state for a random walk $\mathcal{W}_u$, starting at u, on G. (No such expression for digraphs).

First Visit Time Lemma.

Suppose that the connected graph $G = (V, E)$ has n vertices and m edges.

Let $\pi_x = \frac{deg(x)}{2m}$ denote the steady state for a random walk $\mathcal{W}_u$, starting at u, on G.

Let the mixing time T be defined so that

$$\max_{u,x \in V} |P_u^{(t)}(x) - \pi_x| \leq n^{-3}.$$

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$$\max_{u,x \in V} |P_u^{(t)}(x) - \pi_x| \leq n^{-3}.$$

Fix $u, v \in V$. For $s \geq T$ let

$$\mathcal{A}_s(v) = \{\mathcal{W}_u \textit{ does not visit } v \textit{ in } [T, s]\}$$

We try to get a good estimate of $\mathbf{Pr}(\mathcal{A}_s(v))$.

We have

$$\mathbf{Pr}(\mathcal{A}_s(v)) = e^{-(1+o(1))\pi_v s/R_v}.$$

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where R_v is the expected number of visits by $\mathcal{W}_v$ to v in $[0, T]$.

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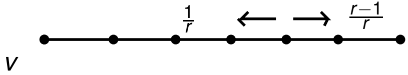
$$\mathbf{Pr}(\mathcal{A}_s(v)) = e^{-(1+o(1))\pi_v s/R_v}.$$

Caveat:
We need $T\pi_v = o(1)$.

Random Regular Graphs

If v is not near any short cycles then

$$R_v \sim \frac{r-1}{r-2}.$$



whp there are very few vertices near short cycles and for these vertices $R_v \leq \frac{r-1}{r-2}$.

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$$
\begin{aligned}
C_u = \mathbf{E} T_G(u) &= \sum_{s>0} \mathbf{Pr}(T_G(u) \geq s) = \sum_{s>0} \mathbf{Pr}(U_s > 0) \\
&\leq \sum_{s>0} \min\{1, \mathbf{E} U_s\} \leq t + \sum_{v \in V} \sum_{s>t} \mathbf{Pr}(\mathcal{A}_s(v))
\end{aligned}
$$

$$
\begin{aligned}
C_u &\leq t + \sum_{v \in V} \sum_{s > t} \mathbf{Pr}(\mathcal{A}_s(v)) \\
&\leq t + n \sum_{s > t} \exp\left\{ -(1 - o(1)) \frac{s(r-2)}{n(r-1)} \right\} \\
&\leq t + \frac{2n^2(r-1)}{r-2} \exp\left\{ -(1 - o(1)) \frac{t(r-2)}{n(r-1)} \right\}
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Taking

$$
t = (1 + o(1)) \frac{r-1}{r-2} n \ln n
$$

we get

$$
C_u \leq (1 + o(1)) t.
$$

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Then let $S(t)$ denote the vertices in S which are not visited by $\mathcal{W}_u$ by time t.

Thus

$$\begin{aligned} \mathbf{E}(|S(t)|) &\geq -T + |S| \exp\left\{-(1-o(1))\frac{t(r-2)}{n(r-1)}\right\} \\ &\to \infty \end{aligned}$$

if $t = (1-o(1))\frac{r-1}{r-2} n \ln n$.

To finish we argue that for $x, y \in S$,

$$\mathbf{Pr}(\mathcal{A}_t(x) \wedge \mathcal{A}_t(y)) \sim \mathbf{Pr}(\mathcal{A}_t(x))\mathbf{Pr}(\mathcal{A}_t(y))$$

and so

$$E(|S(t)|^2) \sim \mathbf{E}(|S(t)|)^2$$

and then the Chebyshev inequality implies that $S(t) \neq \emptyset$ **whp.**

The analysis is valid for regular graphs for which

1. the mixing time T is small and
2. there are few short cycles (or more precisely, for which R_v can be computed easily).

The cover time of $G_{n,p}$

Recall that

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$$R_v = 1 + o(1) \qquad \textit{for all } v \in V$$

and so

$$\mathbf{Pr}(\mathcal{A}_s(v)) \sim e^{-(1+o(1))s\,deg(v)/2m}$$

for $v \in V$.

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Suppose that $k = \alpha \ln n$. There are approximately

$$n \binom{n-1}{k} p^k (1-p)^{n-1-k} \sim n^{1-c+\alpha \ln(ce/\alpha)}$$

vertices of degree k.

The cover time of $G_{n,p}$

Recall that

$$C_u \leq t + \sum_{v \in V} \sum_{s > t} \mathbf{Pr}(\mathcal{A}_s(v))$$

So, if $t = \tau n \ln n$,

$$C_u \leq t + \sum_{\alpha} n^{2 - c + \alpha \ln(ce/\alpha) - \alpha\tau/c + o(1)}$$

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Now

$$\max_{\alpha} 2 - c + \alpha \ln(ce/\alpha) - \alpha\tau/c = 2 - c + ce^{-\tau/c}$$

The cover time of $G_{n,p}$

Recall that

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So,

$$C_u \leq \tau n \ln n + O(n^{2-c+ce^{-\tau/c}+o(1)})$$

and

$$C_u \leq (1+o(1))c \ln \left(\frac{c}{c-1} \right) n \ln n.$$

after putting

$$\tau = (1+o(1))c \ln \left(\frac{c}{c-1} \right).$$

Note that with this value of τ we have

$$2 - c + ce^{-\tau/c} \sim 1.$$

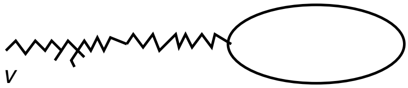
The cover time of $G_{n,p}$

The lower bound is done via Chebyshev, as before.

The Matthews bound works equally well here.

The cover time of the giant component of a sparse random graph

$p = \alpha \ln n/n$ with $0 < \alpha < 1$



$$R_T(1) \sim \ell \text{ and } \mathbf{E}(\#v) \sim n^{1-\alpha\ell+o(1)}$$

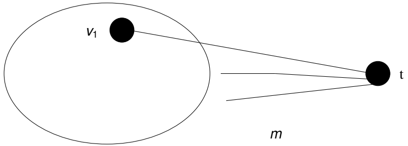
For the cover time choose t such that for all ℓ

$$n^{1-\alpha\ell+o(1)} \exp\left\{-t \cdot \frac{1}{2\alpha n \ln n} \cdot \frac{1}{\ell}\right\} = o(t).$$

Cover time of preferential attachment graph

Sequence of random graphs $G(t)$
$G(t) = G(t-1)$ plus vertex t and m random edges $\{t, v_i\}, i = 1, 2, \ldots, m$.

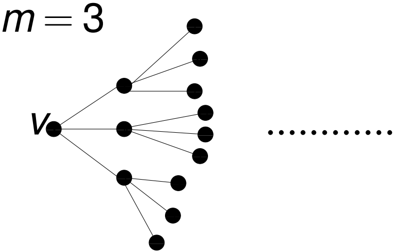
The vertices $v_1, v_2, \ldots, v_m$ are chosen with probability proportional to their degree after step $t-1$.



Whp

$$C_G \sim \frac{2m}{m-1} t \ln t, \qquad \textit{for } m \geq 2.$$

Hardest vertices to cover:



$R_v \sim \frac{m}{m-1}$ and $\pi_v = \frac{m}{2mt}$

This gives that **whp**

$$C_G \sim \frac{2m}{m-1} t \ln t,$$

for $m \geq 2$.

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We assume that $p = \frac{c \ln n}{n}$ where $c - 1$ is at least a constant.

We can use our lemma on $\mathbf{Pr}(\mathcal{A}_s(v))$. It is easy to show that $R_v = 1 + o(1)$ for all vertices. The main difficulty is in establishing the steady state π of a random walk on $D_{n,p}$.

Theorem

Whp

$$\pi_y \sim deg^-(y) \qquad \textit{for all } y \in V$$

where deg^- *refers to in-degree.*

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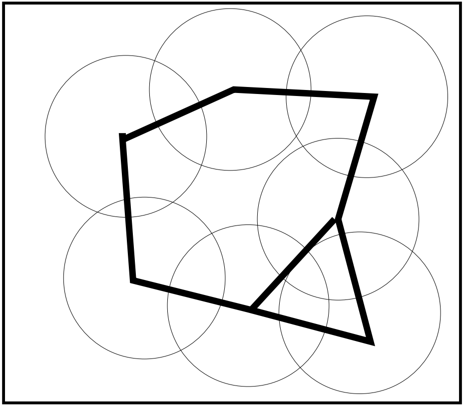
Given the above we can proceed as before to show that **whp**

$$C_D \sim c \ln\left(\frac{c}{c-1}\right) n \ln n.$$

Cover time of random geometric graphs.

Random geometric graph $G = G(d, r, n)$ in d dimensions:
Sample n points V independently and uniformly at random from $[0, 1]^d$. For each point x draw a ball $D(x, r)$ of radius r about x.
$V(G) = V$ and $E(G) = \{\{v, w\} : \ w \neq v, \ w \in D(v, r)\}$



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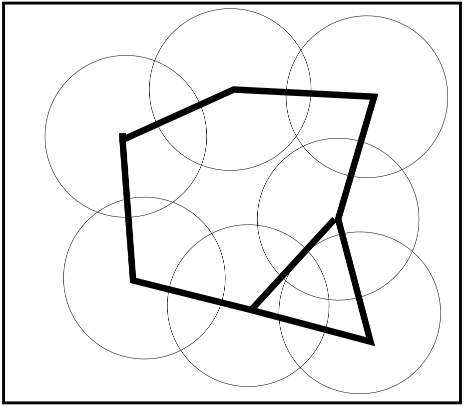

For simplicity we replace $[0,1]^d$ by a torus.

Avin and Ercal $d = 2$

Theorem

$$C_G = \Theta(n \log n) \textbf{ whp}.$$

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Cooper and Frieze $d \geq 3$:

Theorem

Let $c > 1$ be constant, and let $r = \left(\frac{c \log n}{\Upsilon_d n}\right)^{1/d}$. Then **whp**

$$C_G \sim c \log\left(\frac{c}{c-1}\right) n \log n.$$

Υ_d is the volume of the unit ball in d dimensions.

Given the grid Γ it is easy to use a Canonical Paths argument to show that

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When $d \geq 3$ one can show that $R_v = 1 + o(1)$ and then our method works.

Given the grid Γ it is easy to use a Canonical Paths argument to show that

$$T = \tilde{O}(n^{2/d})$$

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All sorts of problems with $d = 2$. Mixing time is relatively large and more important, it has been hard to estimate R_v

Random graphs with a fixed degree sequence.

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Assume that $\nu_2 \gg N$ so that $\xi = \frac{M}{\nu_2 + M} \sim \frac{M}{\nu_2}$. Then

$$T = \Omega\left(\frac{\ln M}{\xi^2}\right).$$

It takes time ξ^{-2} to cross the path replacing a typical kernel edge.

If v has degree two then

$$T\pi_v = \Omega\left(\frac{\ln M}{\xi^2 \nu_2}\right) = \Omega\left(\frac{\nu_2 \ln M}{M^2}\right) \neq o(1) \text{ for large } \nu_2.$$

Solution?

Let

$$\ell^* = \frac{1}{\omega\xi} \text{ where } \omega = N^{o(1)}$$

Replace the path P_e by a path of length ℓ_e by a path of length ℓ_e/ℓ^* to get a graph G^* and then inflate the covertime of G^* by $(\ell^*)^2$.

In G^* we have $T = \tilde{O}(\omega^2)$ and $\pi_v = O(N^{-1+o(1)})$ and so $T\pi_v = o(1)$.

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Problem: We assumed that for all e, ℓ_e was a multiple of ℓ^*. What if $\ell_e = 1$?

To make sense, we have to couple a walk on G with a walk $\mathcal{W}^*$ on G^* where edges in G^* have weight ℓ^*/ℓ_e and the walk chooses edges with probability proportional to weight.

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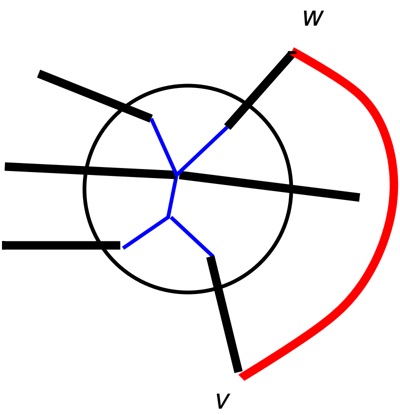
We now do a speedy walk that ignores the time taken to cross small edges. This walk behaves nicely. We can then use concentration of measure to show that the real walk spends relatively little time on the small edges.



The probability of following the red edge at v will be the probability that the walk goes down the black edge incident with v and that w is the first vertex reached, not incident with a small (blue) edge.

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- Analyse random geometric graphs in two dimensions. Some progress made here with Beveridge, Cooper, Mueller and X
- Allow $np = (1 + o(1)) \ln n$ in $D_{n,p}$.
- Tighten results on Crawling on web-graphs. Here the graph grows as the walk progresses and one aims to estimate the proportion of vertices which are unvisited.

Thank You