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1 Common abbreviations for geometry problems

Given triangle ABC:

- a, b, and c are the lengths of the sides opposing vertices A, B, and C, respectively.
- s is the semiperimeter
- r is the inradius
- R is the circumradius

2 Facts, Part I

1. **Extended Law of Sines** $a/\sin A = 2R$.
2. $[ABC] = abc/4R$.
3. (Geometry Revisited, page 3.) Let p and q be the radii of two circles through A, touching BC at B and C, respectively. Then $pq = R^2$.
4. **Ceva** Given triangle ABC. Let $D \in BC$, $E \in CA$, and $F \in AB$. Suppose that:
$$\frac{AF}{FB}\frac{BD}{DC}\frac{CE}{EA} = 1.$$
Prove that AD, BE, and CF are concurrent.
5. **Trig Ceva** Given triangle ABC. Let $D \in BC$, $E \in CA$, and $F \in AB$. Suppose that:
$$\frac{\sin CAD}{\sin DAB}\frac{\sin ABE}{\sin EBC}\frac{\sin BCF}{\sin FCA} = 1.$$
Prove that AD, BE, and CF are concurrent.
6. Prove that the centroid of a triangle lies 2/3 of the way down each median.
7. **Steiner-Lehmus** Let ABC be a triangle such that the lengths of two angle bisectors are equal. Prove that ABC is isosceles.
8. (Geometry Revisited, page 13.) Prove that $abc = 4srR$.
9. (Geometry Revisited, page 13.) Let r_a, r_b, and r_c be the radii of the three excircles of triangle ABC. Prove that $1/r = 1/r_a + 1/r_b + 1/r_c$.
10. **Orthic Triangle** The feet of the altitudes of triangle ABC determine a triangle, called the *orthic triangle*. Prove that the orthocenter of ABC is the incenter of that triangle.
11. **Euler Line** Let O, G, and H be the circumcenter, centroid, and orthocenter of ABC, respectively. Prove that O, G, and H are collinear, and that $HG = 2GO$.