A C^1 Tetrahedral Finite Element Without Edge Degrees of Freedom

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Corrigendum to Lemma 2.2

The first part of the Lemma 2.2, which relates the reduced basis function on a face from the three "unreduced" basis functions, was incorrectly stated. The corrected statement, given next, is used to justify equation (3.1).

Lemma 2.2. *Let t be a triangle with vertices $\{v^{(i)}\}_{i=0}^2$ and let $\{x^{(i)}\}_{i=0}^2 \subset t$ be the points with barycentric coordinates* $(3/5, 1/5, 1/5)$, $(1/5, 3/5, 1/5)$, *and* $(1/5, 1/5, 3/5)$.

- *Let $\{p_i\}_{i=0}^2 \subset \{\mathcal{P}_4(t) \mid p|_{\partial t} = 0\}$ satisfy $p_i(x^{(j)}) = \delta_{ij}$ and $p \in \{\mathcal{P}_3(t) \mid p|_{\partial t} = 0\}$ satisfy $p(c_t) = 1$ where $c_t \in t$ is the centroid. Then*

$$p(x) = (81/125) \left(p_0(x) + p_1(x) + p_2(x)\right).$$

- *If $p \in \{\mathcal{P}_4(t) \mid p|_e \in \mathcal{P}_3(e),\ e \subset \partial t\}$ then $p \in \{p \in \mathcal{P}_3(t) \mid p(c_t) = 0\}$ if and only if*

$$p(x^{(i)}) = (12/25)p(v^{(i)}) - (8/125)\left(p(v^{(j)}) + p(v^{(k)})\right) + (6/125)(dp^i_{ij} - dp^i_{ki}) - (2/125)(dp^j_{jk} - dp^k_{jk}).$$

  where (i, j, k) is an even permutation of $(0, 1, 2)$, and $dp^m_{pq} \equiv \nabla p(v^{(m)}).(v^{(p)} - v^{(q)})$.

Both of these statements can be verified by explicit computation on the parent triangle and then transforming to an arbitrary triangle under an affine map. An alternative proof of the first statement follows upon observing that

$$q(x) \equiv (81/125) \frac{p_0(x) + p_1(x) + p_2(x)}{p(x)} \in \mathcal{P}_1(t).$$

Using the property that $p(x)$ is the unit bubble function, $p(x) = 27\lambda_0(x)\lambda_1(x)\lambda_2(x)$ where $\lambda_i(x)$ are the barycentric coordinates, we compute $p(x^{(i)}) = 81/125$ and

$$q(x^{(i)}) = (81/125) \frac{1}{p(x^{(i)})} = 1, \qquad i = 0, 1, 2.$$

It follows that $q(x) = 1$ is the constant polynomial.

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