Homework Set 6

1) A group is simple if it has no nontrivial proper normal subgroups. Let G be a simple group of order 168. How many elements of order 7 are there in G?

2 Let G be a group of order 231. Prove that the (unique) Sylow 11-subgroup of G is in the center.

3) Prove that for every even integer $p \ge 2$, there exists a constant c(p) such that any one-distance set with respect to the L_p -norm in \mathbb{R}^n has at most $n^{c(p)}$ points.

4) Let λ be a nonzero integer, J be the $m \times m$ all ones matrix, and D be the $m \times m$ diagonal matrix with positive integer diagonal entries $\gamma_1, \ldots, \gamma_m$. Compute the determinant of $A = \lambda J + D$.