Homework Set 1

1) Let R be a ring and $e \in R$ such that $e^2 = e$. Show that $(xe - exe)^2 = (ex - exe)^2 = 0$ for all $x \in R$.

2) Let D be an integral domain with unit of characteristic $p \neq 0$. Prove that p is prime. Extra credit: What if D has no unit?

3) Let D be a commutative ring. Prove that D is an integral domain if and only if the following holds: for every $a, b, c \in D$ with $a \neq 0$, if $ab = ac$ then $b = c$.

4) Let R be a ring in which $x^3 = x$ for every $x \in R$. Prove that R is commutative. Hint: use 1)