False Patterns

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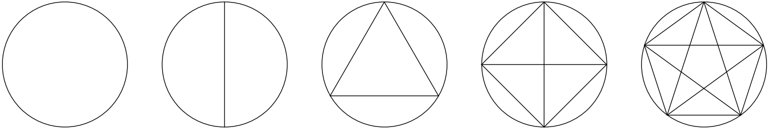
Disk dishonesty

Draw all possible chords between n points in a circle, placed in such a way that no three chords intersect in a single point.

How many regions are formed?



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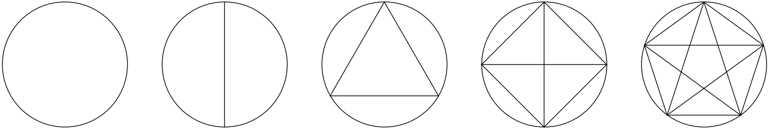
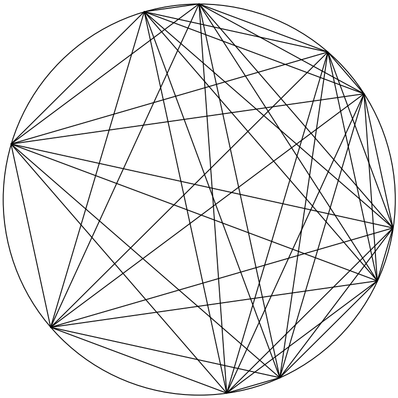


The sequence begins $1, 2, 4, 8, 16, \ldots$

Does this pattern continue?

Are you sure?

Consider the diagram below:



It has 256 regions. (I counted.)

Prime prevarications

Which of these are true?

- All the numbers in this sequence are prime:

$$41 \xrightarrow{+2} 43 \xrightarrow{+4} 47 \xrightarrow{+6} 53 \xrightarrow{+8} 61 \xrightarrow{+10} 71 \xrightarrow{+12} \cdots$$

- If you take a prime number $p \neq 5$, write it in base 5, and reverse the digits, the resulting number is always prime.

$$p = 269 = 2034_5 \quad \Rightarrow \quad 4302_5 = 577 \text{ is prime.}$$

- All of the following numbers are prime:

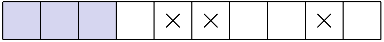
$$2^2 - 1, 2^{2^2-1} - 1, 2^{2^{2^2-1}-1} - 1, 2^{2^{2^{2^2-1}-1}-1} - 1, \ldots$$

Fibonacci falsehoods

All of these sequences begin $1, 2, 3, 5, 8, 13, \ldots$.

Which ones are the Fibonacci sequence?

- Let x_n be the number of ways to write n as an ordered sum of odd integers. The 5 ways to write 5 are $5 = 1+1+3 = 1+3+1 = 3+1+1 = 1+1+1+1+1$.
- Let $y_1 = 1$ and y_n be the least number such that all pairwise sums $y_i + y_j$, $i \neq j$, are distinct.
- Let $z_n = \left\lceil e^{\frac{n-1}{2}} \right\rceil$.
- Let w_n be the number of ways to take a grid of n cells, shade in some of the initial cells, and mark an equal number of the remaining cells.



Digit deception

We have

$$\sum_{n=0}^{\infty} \frac{\lfloor n\cos(1+\frac{1}{21})\rfloor}{2^n} = 0.333\,333\,333\,333\,333\,333\,333\ldots$$

Is this sum actually $\frac{1}{3}$? If not, for how many digits does the pattern continue?

What about

$$\sum_{n=1}^{\infty} \frac{\lfloor 5^{1/4} n\rfloor}{3^n} = 0.812\,499\,999\ldots?$$

Big lies

Define Big(n) to be the number of times the digits 5, 6, 7, 8, 9 occur in the decimal expansion of n.

(For example, Big(2016) $= 1$ and Big(1048576) $= 4$.)

Big lies

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(For example, $\mathrm{Big}(2016) = 1$ and $\mathrm{Big}(1048576) = 4$.)

Is it true that:

$$\sum_{n=0}^{\infty} \frac{\mathrm{Big}(n)}{2^n} = \frac{2}{33}?$$

$$\sum_{n=0}^{\infty} \frac{\mathrm{Big}(n)}{n(n+1)} = \frac{2}{9} \log 2?$$

(Both are accurate to at least 15 decimal places.)

Reputable references

- Borwein, J. M.; Borwein, P. B. Strange series and high precision fraud. *Amer. Math. Monthly* **99** (1992), no. 7, 622–640.
- Guy, Richard K. The strong law of small numbers. *Amer. Math. Monthly* **95** (1988), no. 8, 697–712.
- Guy, Richard K. The second strong law of small numbers. *Math. Mag.* **63** (1990), no. 1, 3–20.
- *The On-Line Encyclopedia of Integer Sequences*, published electronically at `https://oeis.org`.