Warm-up

1. (AMC 10 2008) Let $k = 2008^2 + 2^{2008}$. What is the units digit of $k^2 + 2^k$?

Problems

1. How many divisors does 10000 have? (That is, how many numbers between 1 and 10000 divide 10000 evenly?)

2. How many of the numbers between 1 and 100 have exactly 3 divisors?

3. Find the smallest integer with exactly 10 divisors.

4. What is the sum of all the divisors of 10000? (Try to find a way to do this without actually adding up a bunch of numbers.)

5. If the sum of all of the divisors of $n$ (including $n$ itself) is 91, what is $n$?

6. For what values of $n$ will a regular $n$-sided polygon have angles whose measure (in degrees) is an integer?

7. A triple of positive integers $(x, y, z)$ is called a Pythagorean triple if $x^2 + y^2 = z^2$. Find all Pythagorean triples where $x = 8$ or $x = 9$. (Don’t assume that $x < y$.)

8. Find all pairs of numbers $(x, y)$ such that the GCD of $x$ and $y$ is 12, and the LCM of $x$ and $y$ is 180.

9. Find all pairs of numbers $(x, y)$ such that the GCD of $x$ and $y$ is 12, and the product of $x$ and $y$ is 180.

10. Find the number of zeroes at the end of $100! = 1 \times 2 \times 3 \times \cdots \times 100$. 