21-268 – Homework assignment week #8

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Reminder

Homework is due next Wednesdays before 5pm, to me in class or in Christopher Cox's mailbox in Wean Hall 6113 (pay attention to the arrow !). Late homework will never be accepted without a proper reason. In case of physical absence, electronic submissions by e-mail to both me your TA can be accepted. Please do not forget to write your name, andrew id and section and please use a staple if you have several sheets.

Exercises (24 pts)

1. (6 pts) Compute the integral

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{1}(x^2+y^2+z^2)dzdydx.$$

   by changing to spherical coordinates.

2. (6 pts) Let R be the parallelogram in the x, y plane with vertices $(0,0)$, $(2,1)$, $(1,2)$ and $(3,3)$. Compute the integral

$$\iint_R (2x^2-5xy+2y^2)dA_{xy}.$$

   Note: computing this integral directly in the form given would be very long and is not what is intended.

3. (2+2+2 pts) Let $x(u,v)=uv$ and $y(u,v)=u+v$. Let $R_{uv}=\{(u,v): 2\leq u\leq 3 \text{ and } 0\leq v\leq 1\}$ and define $R_{xy}=\{(x(u,v),y(u,v)) : (u,v)\in R_{uv}\}$. Prove that this mapping from R_{uv} to R_{xy} is one to one by solving for (u,v) given (x,y). Make a drawing of R_{xy}. Compute

$$\frac{\partial(x,y)}{\partial(u,v)}.$$

   Also compute

$$\int\int_{R_{xy}} y dA_{xy}.$$

4. (6 pts) Let $R_{xy}=\{(x,y): 5x^2+2xy+2y^2\leq 1\}$. Use the transformation $x=u+v$, $y=-2u+v$ to compute

$$\int\int_{R_{xy}} \sqrt{5x^2+2xy+2y^2}dA_{xy}.$$