21-241 – Homework assignment week #8

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Reminder

Homework will be given on Fridays and due on the next Friday before 5pm, to me in class or in Andrew Zucker's mailbox in Wean Hall 6113. Late homework will never be accepted without a proper reason. In case of physical absence, electronic submissions by e-mail to both me and Andrew Zucker can be accepted. Please do not forget to write your name, andrew id and section and please use a staple if you have several sheets.

Reading

- Poole: section 4.1 and 4.2.

Exercises (18 pts)

Exercise 1 (4+1 pts)

In this exercise we prove a property that we will use in the proof of Theorem 4.8. This of course, prevents us from using Theorem 4.8 in this exercise.

1. Let A, B be $n \times n$ matrices. Show that

$$AB \text{ is invertible} \Rightarrow A \text{ and } B \text{ are invertible.}$$

   Hint: show first by contraposition that B is invertible and then show directly that A is.

2. Deduce that if A is not invertible then neither is AB.

Exercise 2 (2+2 pts)

Let $A, B \in \mathcal{M}_{nn}(\mathbb{R})$. A is called *idempotent* if $A^2 = A$. B is called *nilpotent* if there exists some $m \geq 1$ such that $B^m = 0$.

1. Find all possible values of $\det(B)$ when B is nilpotent.
2. Find all possible values of $\det(A)$ when A is idempotent.

Exercise 3 (4 pts)

Find all values of $k \in \mathbb{R}$ such that

$$A = \begin{bmatrix} k & k & 0 \\ k^2 & 2 & k \\ 0 & k & k \end{bmatrix}$$

is invertible.

Exercise 4 (3+2 pts)

Back to eigenvalues

1. Use the determinant to compute the eigenvalues of

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

2. Use row reduction to compute the associated eigenspaces.