21-124 MODELING WITH DIFFERENTIAL EQUATIONS

LECTURE 12: NULLCLINES

In the twelfth lecture, we discussed a method for analyzing the behavior of a system of equations which did not rely on computer technology. We introduced the concept of *nullclines*, and showed how they could be used to predict the behavior of a system.

1. NULLCLINES

We have found in the past that the equilibrium points of the system

$$\frac{dx}{dt} = f(x,y)$$
$$\frac{dy}{dt} = g(x,y)$$

could be found by solving the system of equations

$$f(x,y) = 0$$
$$g(x,y) = 0.$$

Wherever $f(x,y) = 0$, x is not changing, and wherever $g(x,y) = 0$, y is not changing. If both of these are satisfied at the same time, neither variable is changing and the system is in equilibrium.

Today we look separately at the two sets of points where $f(x,y) = 0$ and $g(x,y) = 0$ separately.

The curves in the xy-plane where $f(x,y) = 0$ are called x*-nullclines.* At those points, $\frac{dx}{dt} = 0$, and so there is no change in the variable x. Consequently, any solution that passes through one of those points travels vertically – either up or down. The curves defined by $f(x,y) = 0$ will divide the plane into a number of different regions. On some of these regions $\frac{dx}{dt}$ will be positive (so solutions will be moving toward the right). On some of them $\frac{dx}{dt}$ will be negative (with solutions moving to the left).

A similar analysis can be made for the y-nullclines, i.e. the curves defined by $g(x,y) = 0$. Solutions will cross these curves horizontally, traveling either to the left or right. The y-nullclines divide the plane into regions where solutions are traveling either up or down.

Now, the collection of both x- and y-nullclines will divide the plane into regions, and each can be labeled in one of four ways: UR, wherin solutions travel up and to the right; DL, wherin solutions travel down and to the left; UL (up-left); and DR (down-right).

The points where x-nullclines intersect y-nullclines are equilibrium points.

All of this information taken together can be used to sketch solutions to the system.