21-124 MODELING WITH DIFFERENTIAL EQUATIONS

LECTURE 11: A MASS–SPRING SYSTEM IN TWO DIMENSIONS
EXPERIMENTS AND DISCUSSION

In Lecture 10 we built a mathematical model of a mass on a spring that is free to move in a vertical plane. We then modified the model to account for an external force applied by moving the point of attachment of the spring in a small vertical circle. The function M-file we developed is called `movingspring.m` and is available on the 21-124 website.

Today we will continue to investigate this system. We will develop a graph which shows the maximum displacement from the equilibrium point of the static system as a function of the forcing frequency. We will look for peaks of this graph, which indicate resonance, and investigate this graph in more detail near these points.

1. Use `ode45` with the values $k = .7$, $m = 1.2$ and $\omega = 1$ to compute a solution to the "movingspring" system with initial condition $[0, 0, -17.8, 0]$ on the time interval $[0, 50]$.

2. Determine the maximum displacement (in the xy-plane) of the solution from the point $(0, -17.8)$.

3. Write a script M-file that will, for a given value of ω
   (a) Compute a solution with initial condition $[0, 0, -17.8, 0]$ on the time interval $[0, 50]$, and
   (b) Find the maximum displacement of the solution from the point $(0, -17.8)$.

4. Now, modify the script you wrote so that it will, for a range of values of omega,

(a) Compute for each value of ω a solution with initial condition $[0, 0, -17.8, 0]$ on the time interval $[0, 50]$,
(b) Find the maximum displacement of each solution from the point $(0, -17.8)$,
(c) Keep track of the forcing frequencies, and corresponding amplitudes for each solution, and
(d) Plot a curve of maximum displacement as a function of the forcing frequency, (i.e. a frequency response curve).

You may want to start with a script that tries only a small number of values for ω until you have it working correctly, say all the values between .1 and 1, stepping by .1. It may help to use a `for` loop in your script:

```
for omega=.1:.1:1
<statements>
end
```

5. Use the script file you have created to look at the frequency response curve. Look for peaks in the curve that indicate resonance. How the peak(s) you find compare to the natural frequency for the one-dimensional oscillator with $m = 1.2$ and $k = .7$ ($\omega_0 = .7637$)?

   Due to computing time issues, if you increase the number of values of omega you consider, you may have to shorten the interval on which solutions are computed. There are some general principles to consider: If you allow ω to take values in a broader range, you get a more complete view of the situation. If you reduce the size of the steps that ω takes, you will get a smoother looking curve. If you increase the length of the interval on which you compute the solutions, you get a better measure of the maximum amplitude achieved. Finally all three of these adjustments tend to increase the number of operations performed, and hence computing time.

6. Look more closely at the frequency response in the region near a peak. (Here you can trade off a smaller range of ω's for a reduction in the step between ω's. You may also want to increase the interval of the solutions a bit.

   Does the response curve have a simple maximum here, or is there something more complicated happening?