21-124 MODELING WITH DIFFERENTIAL EQUATIONS

HOMEWORK 6: DUE ON FRIDAY, APRIL 12 BY 5:00PM

In the last few lectures, we have looked at a mass-spring system that is free to move in a vertical plane. In this problem set, we will examine a mass-spring system that moves in a horizontal plane. In this case, gravity is not an issue. We will use the same mass and spring as in the lectures: the mass is 1.2 kg, the spring constant is .7 N/m, and the spring has an equilibrium length of 1 m. One end of the spring is fixed to a point that rotates around the origin, with coordinates $[cos(\omega t), sin(\omega t)]^T$. The other end is attached to the mass is free to move about the plane.

1. (a) Derive a system of second order equations that describe the motion of the mass. You can use the notes from Lecture 10 and the M-file `movingspring.m` as a guide.
   (b) Convert this system of second order equations to a first order system.
   (c) Write a function M-file that can be used with `ode45` to compute solutions to this system. Compute the solution with $\omega = 1$, and initial conditions $x = 1.1$, $y = 0$ with the mass initially at rest. The solution should be computed on the interval $t \in [0, 30]$. Graph the motion of the mass in the xy-plane.

2. (a) Write a script M-file (similar to the one you used in Lecture 11) that will produce a frequency response curve. The graph should show the amplitude (distance from the origin) of solutions with the above initial conditions, and computed on the interval $[0, T]$ with $T \geq 50$. Show the response for the values $0 \leq \omega \leq 3$.
   (b) Choose a smaller interval that includes a peak of the graph you produced above. Produce a more detailed response curve in this region - compute the solutions for a longer period of time, and/or compute solutions for a larger number of values of ω.

3. Choose a value of ω that results in a large amplitude, and a value which results in a small amplitude. Plot the graphs that show the motion (in the xy-plane) for each of these values. In what ways are they similar? In what ways are they different?