

21-124 MODELING WITH DIFFERENTIAL EQUATIONS

HOMEWORK 4: DUE IN CLASS ON MARCH 12

1. Consider the system

$$\begin{aligned}\frac{dP}{dt} &= \alpha P \left(1 - \frac{P}{\beta}\right) - \gamma PQ \\ \frac{dQ}{dt} &= \delta Q \left(1 - \frac{Q}{\epsilon}\right) - \zeta PQ\end{aligned}$$

where $\alpha = 2$, $\beta = 7$, $\gamma = 1$, $\delta = 6$, $\epsilon = 9$, $\zeta = 2$. which represents the growth of two populations of animals (Pigeons and Quails, perhaps).

- What are the assumptions on which this model is based?
- Describe the roll played by each of the parameters. (The six greek letters.)
- How would you describe the interaction of these two populations? Are they competing or cooperating? Is it a predator/prey relationship? Is there some other type of interaction?
- What are the equilibrium points of the system? (Compute them by hand.)
- Use pplane to produce the phase plane for the system. Experiment by plotting a bunch of solutions. How many non-equilibrium solutions can you find? Turn in a printout showing one or two of each type. (Enough to give an idea what is going on, but not so many that it looks crowded and confusing.)
- What does this model predict for the future of these two populations? Does it depend on the initial conditions? If so, How?

2. Consider the system

$$\begin{aligned}\frac{dP}{dt} &= \alpha P \left(1 - \frac{P}{\beta}\right) + \gamma PQ \\ \frac{dQ}{dt} &= \delta Q \left(1 - \frac{Q}{\epsilon}\right) + \zeta PQ\end{aligned}$$

where $\alpha = 2$, $\beta = 2$, $\gamma = 0.1$, $\delta = 6$, $\epsilon = 3$, $\zeta = 0.2$. which represents the growth of two populations of animals (Pelicans and Q...?).

- What are the assumptions on which this model is based?
- Describe the roll played by each of the parameters. (The six greek letters.)
- How would you describe the interaction of these two populations? Are they competing or cooperating? Is it a predator/prey relationship? Is there some other type of interaction?
- What are the equilibrium points of the system? (Compute them by hand.)
- Use pplane to produce the phase plane for the system. Experiment by plotting a bunch of solutions. How many non-equilibrium solutions can you find? Turn in a printout showing one or two of each type. (Enough to give an idea what is going on, but not so many that it looks crowded and confusing.)

- (f) What does this model predict for the future of these two populations? Does it depend on the initial conditions? If so, How?

3. Consider the system

$$\begin{aligned}\frac{dP}{dt} &= 5P \left(1 - \frac{P}{2}\right) - PQ - h \\ \frac{dF}{dt} &= -Q + 2PQ\end{aligned}$$

which represents the growth of two populations of animals, one of which preys upon the other. The parameter h represents hunting at a constant rate.

- What are the assumptions on which this model is based? Which is the predator, and which is the prey?
- What are the equilibrium points of the system? (Compute them by hand. Do they depend on h ? If so, how?)
- Use pplane to produce the phase plane for the system. Experiment by plotting a bunch of solutions. How many non-equilibrium solutions can you find? Try this for several different values of h between 0 and 0.5. How does changing h affect the equilibrium points?
- Choose two values of h , and turn in a printout for each. The printouts should show several different types of solution.
- What does this model predict for the future of these two populations? Does it depend on the initial conditions? If so, How?
- How can you use this model to explain the increase in the shark population in the Mediteranean Sea during World War II? (Hint: Sharks eat the same fish people do, and naval warfare tends to decrease the size of fishing fleets.)