Discrete Mathematics 21-228
Assignment #9
Due Wednesday, June 23

1. Let G be a graph with n vertices. If M is a maximum matching of G containing m edges, prove that G has an independent set of size $n - 2m$.

2. A graph G is called *d-regular* if every vertex has degree d. Let $G = (V, E)$ be a d-regular bipartite graph with bipartition $V = A \cup B$.

   (a) Prove that $|A| = |B|$.

   (b) Use the Marriage theorem to prove that for $d \geq 1$ then G has a perfect matching.

   (c) Prove that there exists a partition of E into sets $M_1, \dots, M_d$ such that M_i is a perfect matching for $i = 1, \dots, d$.

3. A graph G has 7 connected components and no odd cycles. How many different ways are there to properly color the vertices black and white?

4. (a) If G is the Peterson Graph, use results proven in class to show that $3 \leq \chi(G) \leq 4$.

   (b) Let G_n be the graph arising from K_n (the complete graph on n vertices) by omitting the edges of a Hamilton cycle. Determine the chromatic number of G_n.

5. Show that a tree has at most one perfect matching.

6. Let $G = (V, E)$ be a bipartite graph with bipartition $V = A \cup B$, such that $|A| = |B| = m$. Prove that if the degree of every vertex is larger than $m/2$ then G has a perfect matching.