Discrete Mathematics 21-228
Assignment #8
Due Friday, June 18

1. Let $G = (V, E)$ be a graph.
   (a) Prove:
   $$\sum_{v \in V} deg(v) = 2|E|$$
   (b) Prove that the number of vertices of odd degree in a graph is even.
2. For which values of m, n does $K_{m,n}$ have a Hamilton cycle?
3. Prove that if G has exactly n vertices and at least n edges then it must contain a cycle.
4. Let $G = (V, E)$ be the graph with $V = \{1, 2, 3, 4\}, E = \{12, 23, 24, 34\}$. Let $A(G)$ be the adjacency matrix of G, and $M(G)$ be the incidence matrix. Recall that the transpose of an m x n matrix B is the n x m matrix B^T, defined by $B^T(i, j) = B(j, i)$. Calculate AA^T, MM^T, and $M^T M$. What does the (i, j) entry in each matrix signify?
5. (a) Give an example of a graph G where both G and $\overline{G}$ are connected.
   (b) Let G be a graph. Prove that either G or $\overline{G}$ is connected.