Discrete Mathematics 21-228
Assignment #6
Due Thursday, June 10

1. You are dealt a poker hand: a set of five cards drawn at random from a standard deck. Describe the probability space for this experiment. Determine the probabilities for the following events:

   (a) You are dealt a flush (all cards are of the same suit).

   (b) You are dealt four of a kind (4 cards of the same value, with one other card, e.g {4, 4, 4, 4, 9})

   (c) You are dealt three of a kind (3 cards of the same value, with 2 other cards of different values which are also different from each other, e.g {4, 4, 4, Q, 9})

   (d) You are dealt a full house (3 cards of the same value, with 2 other cards which have the same value as each other, e.g {4,4,4,Q,Q})

2. Alice and Bob play a game in which they alternately toss a pair of fair dice. The one who is first to roll a total of 7 wins the game. What is the probability that the person who goes first wins the game?

3. You are given a (fair) 6-sided die. To win a prize, you can either try to:

   (a) roll at least one 6 in 6 rolls.

   (b) roll at least 2 6's in 12 rolls.

   (c) roll at least 3 6's in 18 rolls.

   Which option should you choose?

4. Two identical-looking coins are placed in front of you. You are told that one of the coins is fair, and the other comes up heads 3/4 of the time. You pick up one of the coins and flip it 5 times, getting 3 tails and 2 heads. What is the probability you picked up the fair coin? Hint: Bayes' Theorem.

5. Let $X$ and $Y$ be Poisson random variables with parameters $\lambda_1$ and $\lambda_2$, respectively. What is $Pr(X + Y = n)$? (You may assume the events $\{X = i\}$ and $\{Y = j\}$ are independent.) Hint: Binomial Theorem.