Discrete Mathematics 21-228
Assignment #4
Due Thursday, May 27

1. How many bit strings of length n have an even number of zeros?

2. How many strings of length n on the digits $\{0, 1, 2\}$ have no two consecutive digits identical?

3. Solve the following recurrence:

$$a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$$

for $n \geq 3$, with initial conditions $a_0 = 6, a_1 = 6$, and $a_2 = 18$.

4. The Tower of Hanoi puzzle was invented by the French mathematician Edouard Lucas in 1883. We are given a tower of disks, initially stacked in increasing size on one of three pegs. The objective is to transfer the entire tower to one of the other pegs, moving only one disk at a time and never a larger one onto a smaller one. You can do this puzzle online at: http://thinks.com/java/hanoi/hanoi.htm
Let H_n denote the number of moves required to solve this puzzle with n disks. Set up a recurrence relation for H_n, and explain why your recurrence is correct. Solve the recurrence.

5. Use generating functions to count the number of solutions to

$$y_1 + y_2 + y_3 + y_4 = 16$$

where $y_1, y_2, y_3, y_4 \in \mathbf{Z}$ and $0 \leq y_1 \leq 4, 0 \leq y_2 \leq 6, 0 \leq y_3 \leq 5, 0 \leq y_4 \leq 6$